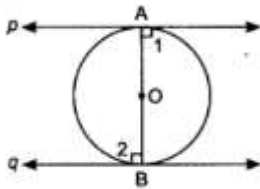




22. (a)  $6 = 2 \times 3$  1/2  
 $72 = 2^3 \times 3^2$  1/2  
 $120 = 2^3 \times 3 \times 5$  1/2  
HCF =  $2^1 \times 3^1 = 6$  1/2  
LCM =  $2^3 \times 3^2 \times 5 = 360$  1/2  
OR

- (b) Time taken by cyclists = 30, 40, 48 minutes 1/2  
Identify LCM 1  
LCM of (30, 40, 48) = 240 minutes 1/2  
∴ They will meet again after 240 minutes (4 hours).

23.



AB is a diameter of the circle, p & q are two tangents,  $OA \perp p$  and  $OB \perp q$  (radius is perpendicular to tangent through the point of contact) 1/2

Therefore  $\angle 1 = \angle 2 = 90^\circ$  1/2

$\Rightarrow p \parallel q$  ( $\angle 1$  and  $\angle 2$  are alternate interior angle) 1/2

24.  $\sin A \cos B + \cos A \sin B = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$  1/2

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$
1/2

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$
1/2

$$= \frac{\sqrt{2}(\sqrt{3}+1)}{4}$$
1/2

OR

$$\sin^2 30^\circ + \cos^2 45^\circ - \cos 0^\circ \times \tan 45^\circ$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 - 1 \times 1$$
1

$$= \frac{1}{4} + \frac{1}{3} - 1$$
1/2

$$= \frac{1+3-4}{12} = \frac{-1}{12}$$
1/2

25. Let the integers be x and x+1 1/2

$$x(x+1) = 306$$

$$x^2 + x - 306 = 0$$
1/2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \frac{1}{2}$$

$$x = \frac{-1 \pm \sqrt{1^2 + 1224}}{2}$$

$$x = \frac{-1 \pm 35}{2} = 17 \quad \frac{1}{2}$$

Therefore the integers are  
17 and 18

26. Integers between 50 and 500 divisible by 7: 56, 63, ..., 497.  $\frac{1}{2}$

$$n = \frac{[(497 - 56)]}{7} + 1 = 64. \quad \frac{1}{2}$$

$$\text{Sum} = \frac{n}{2} (a + a_n) \quad \frac{1}{2}$$

$$= \frac{64}{2} \times (56 + 497) \quad \frac{1}{2}$$

$$= 32 \times 553 \quad \frac{1}{2}$$

$$\therefore \text{Sum} = 17,696. \quad \frac{1}{2}$$

27. Assume  $\sqrt{2} - \sqrt{5}$  is rational = r.  $\frac{1}{2}$

$$\text{Then } \sqrt{2} = r + \sqrt{5}$$

$$\Rightarrow \sqrt{2} - r = \sqrt{5} \quad \frac{1}{2}$$

Squaring both sides:

$$2 - 2r\sqrt{2} + r^2 = 5 \Rightarrow -2r\sqrt{2} = 3 - r^2. \quad \frac{1}{2}$$

$$\Rightarrow \sqrt{2} = \frac{r^2 - 3}{2r} \quad \frac{1}{2}$$

RHS is rational ( $\frac{\text{integer}}{\text{integer}}$ )  $\frac{1}{2}$

LHS =  $\sqrt{2}$ , rational.  $\frac{1}{2}$

LHS  $\neq$  RHS contradiction.

$\therefore \sqrt{2} - \sqrt{5}$  is irrational

28. LHS =  $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2$   $\frac{1}{2}$

$$= \sin^2 \theta + 2 \sin \theta \operatorname{cosec} \theta + \operatorname{cosec}^2 \theta + \cos^2 \theta + 2 \sec \theta \cos \theta + \sec^2 \theta \quad \frac{1}{2}$$

$$= \sin^2 \theta + 2 + \operatorname{cosec}^2 \theta + \cos^2 \theta + 2 + \sec^2 \theta \quad \frac{1}{2}$$

$$= 4 + (\sin^2 \theta + \cos^2 \theta) + (\sec^2 \theta + \operatorname{cosec}^2 \theta) \quad \frac{1}{2}$$

$$= 5 + (1 + \tan^2 \theta + 1 + \cot^2 \theta) \quad \frac{1}{2}$$

$$= 7 + \tan^2 \theta + \cot^2 \theta.$$

Hence proved.

29. 

CI	f	cf
65 - 85	7	7

 $\frac{1}{2}$

85 - 105	8	15
105 - 125	7	22
125 - 145	20	42
145 - 165	14	56
165 - 185	9	65
185 - 205	5	70

Median class is 125-145

1/2

$$\begin{aligned} \text{Median} &= l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h \\ &= 125 + \frac{35 - 22}{20} \times 20 \\ &= 125 + 13 \\ &= 138 \end{aligned}$$

1/2

1/2

1/2

1/2

30. Given  $\angle QPR = 40^\circ$  (angle between tangent and chord PQ).

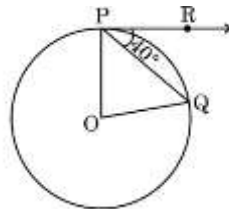
1

$$\angle POQ = 2 \times \angle QPR = 80^\circ.$$

$$\therefore \text{Measure of } \angle POQ = 80^\circ$$

1

1



**OR**

$AP = BP$  (tangents from external point P)

1/2

$\angle PAB = \angle PBA$  (angles opposite to equal sides)

1/2

$$\angle APB + \angle PAB + \angle PBA = 180^\circ$$

1/2

$$\angle PAB = 60^\circ$$

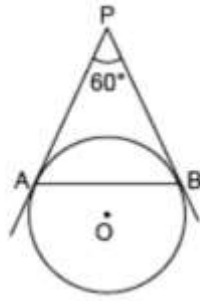
1/2

$\triangle APB$  is an equilateral triangle.

1/2

$$AB = AP = 5 \text{ cm}$$

1/2



31. Let original length =  $x$ , breadth =  $y$ . 1/2

$$(x-5)(y+3) = xy - 9 \dots(1)$$

$$(x+3)(y+2) = xy + 67 \dots(2)$$

Expanding:

$$xy + 2x + 3y + 6 = xy + 67 \Rightarrow 2x + 3y = 61 \dots(A)$$

$$xy - 5y + 3x - 15 = xy - 9 \Rightarrow -5y + 3x = 6 \dots(B)$$

Solving (A) and (B): From (B),  $3x = 5y + 6$

$$\Rightarrow x = \frac{5y+6}{3}$$

$$\text{Substitute in (A): } \frac{2(5y+6)}{3} + 3y = 61$$

$$\Rightarrow 19y + 12 = 183 \Rightarrow y = 9.$$

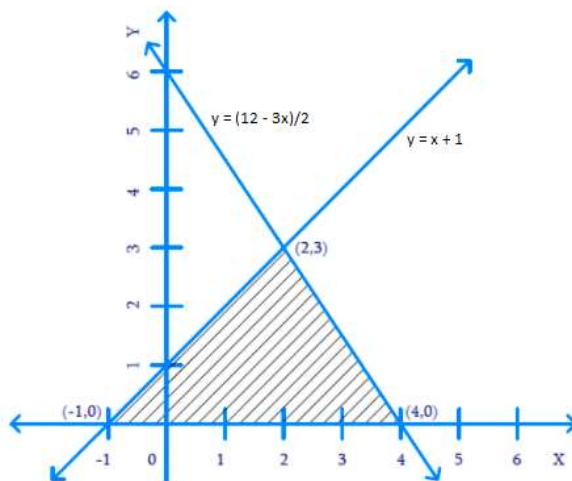
$$\text{Then } x = \frac{5 \times 9 + 6}{3} = \frac{51}{3} = 17.$$

$\therefore$  Dimensions: length = 17 units, breadth = 9 units.

**OR**

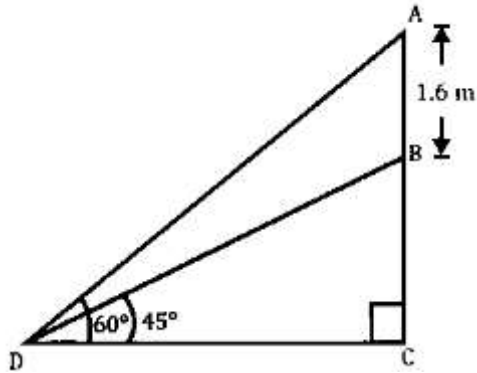
Solutions of  $x - y + 1 = 0$  are  $(-1,0)$ ,  $(2,3)$  and  $(2,2)$

And solutions of  $3x + 2y - 12 = 0$  are  $(0,6)$ ,  $(2,3)$  and  $(4,0)$



Vertices of the triangle are  $(2,3)$ ,  $(4,0)$  and  $(-1,0)$ .

32.



In  $\triangle BCD$ ,

$$\tan 45^\circ = \frac{BC}{CD}$$

$$1 = \frac{BC}{CD}$$

Thus,  $BC = CD$

In  $\triangle ACD$ , In  $\triangle ACD$ ,

$$\tan 60^\circ = \frac{AC}{CD}$$

$$\tan 60^\circ = \frac{AB+BC}{CD}$$

$$\sqrt{3} = \frac{1.6+BC}{BC} \text{ [Since } BC = CD]$$

$$\sqrt{3} BC = 1.6 + BC$$

$$\sqrt{3} BC - BC = 1.6$$

$$BC (\sqrt{3} - 1) = 1.6$$

$$BC = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{1.6(\sqrt{3}+1)}{2}$$

$$= 0.8(\sqrt{3}+1)$$

Height of pedestal  $BC = 0.8(\sqrt{3}+1) \text{ m.} = 2.185$

1

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

33.(A) The volume of the cuboidal portion of the shed is given by the formula

$$V_{\text{cuboid}} = \text{length} \times \text{width} \times \text{height}$$

$$= 14 \times 20 \times 7 = 1960 \text{m}^3$$

$$\text{Volume of the half cylinder} = \frac{1}{2} \pi r^2 h = \frac{1}{2} \pi \times 7^2 \times 20 = 1540 \text{m}^3$$

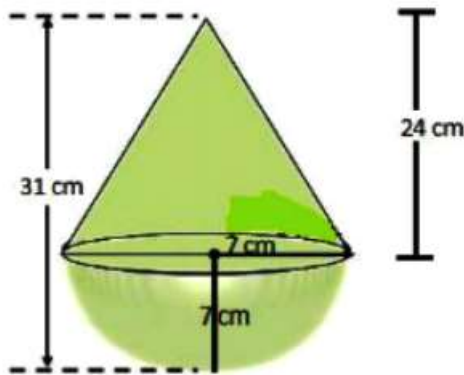
$$\text{Total volume} = 1960 + 1540 = 3500 \text{m}^3$$

$$\text{Space occupied by machinery} = 400 \text{m}^3$$

$$\text{Therefore remaining space} = 3500 - 400 = 3100 \text{m}^3$$

OR

(B)



Total height of the toy is 31 cm.

Radius of hemisphere = 7 cm.

∴ The height of the cone is  $h = 31 - 7 = 24$  cm.

Radius of cone is  $r = 7$  cm.

$$\begin{aligned} \therefore \text{The slant height of the cone is } l &= \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} \\ &= \sqrt{625} = 25 \text{ cm.} \end{aligned}$$

Total surface area of toy = surface area of hemisphere + surface area of cone.

$$= 2\pi r^2 + \pi r l = \pi r (1 + 2r)$$

$$= \frac{22}{7} \times 7 (2 \times 7 + 25)$$

$$= 22 \times (14 + 25)$$

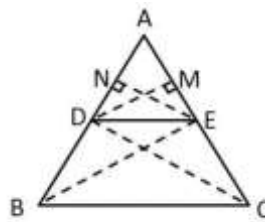
$$= 22 \times 39 = 858 \text{ cm}^2.$$

34. Distance = 300 km, speed =  $x$  km/h, 1/2  
time =  $\frac{300}{x}$  hours. 1/2
- When speed increased by 5 km/h, time =  $\frac{300}{(x+5)}$  hours, 2 hours less. 1/2
- $$\frac{300}{x} - \frac{300}{(x+5)} = 2$$
- 1
- $$\Rightarrow \frac{300(x+5)}{x(x+5)} - \frac{300x}{x(x+5)} = 2$$
- 1/2
- $$\Rightarrow 2x^2 + 10x - 1500 = 0$$
- 1/2
- $$\Rightarrow x^2 + 5x - 750 = 0$$
- 1/2
- Solving  $x = 25$  (since speed can't be negative) 1/2
- $\therefore$  Original speed = 25 km/h. 1/2

OR

- Let incomes =  $9x$ ,  $7x$  and expenditures =  $4y$ ,  $3y$ . 1/2+1/2  
Savings =  $9x - 4y = 2000$ ,  $7x - 3y = 2000$ . 1/2  
Solving:  $(9x - 4y) - (7x - 3y) = 0$  1/2  
 $\Rightarrow 2x - y = 0$  1/2  
 $\Rightarrow y = 2x$ . 1/2  
Substitute in first:  $9x - 8x = 2000$  1/2  
 $\Rightarrow x = 2000$ . 1/2  
 $\therefore$  Incomes:  $9x = ₹18,000$ ,  $7x = ₹14,000$ . 1/2 + 1/2

35. Theorem: A line drawn parallel to one side of a triangle divides the other two sides in the same ratio.  
Proof:



- Consider  $\triangle ABC$ , line  $DE \parallel BC$  intersects  $AB$ ,  $AC$  at  $D$  and  $E$ . 1/2  
Draw  $CE \parallel AD$  meeting  $BC$  at  $F$ .

- In  $\triangle ADE$  and  $\triangle DBE$  1/2

$$\text{Ar}(\triangle ADE) = \frac{1}{2} \times AD \times EN$$

$$\text{Ar}(\triangle DBE) = \frac{1}{2} \times DB \times EN \quad \frac{1}{2}$$

$$\frac{\text{Ar}(\triangle ADE)}{\text{Ar}(\triangle DBE)} = \frac{AD}{DB} \text{-----}(1) \quad \frac{1}{2}$$

Also in  $\triangle ADE$  and  $\triangle ECA$

$$\text{Ar}(\triangle ADE) = \frac{1}{2} \times AE \times DM \quad \frac{1}{2}$$

$$\text{Ar}(\triangle ECD) = \frac{1}{2} \times EC \times DM \quad \frac{1}{2}$$

$$\frac{\text{Ar}(\triangle ADE)}{\text{Ar}(\triangle ECD)} = \frac{AE}{EC} \text{-----}(2) \quad \frac{1}{2}$$

But  $\triangle DBE$  and  $\triangle ECD$  are on the same base and between the same parallels. 1/2

Hence  $\text{ar}(\triangle DBE) = \text{ar}(\triangle ECD)$ -----(3) 1

Therefore, from (1), (2) and (3),  $\frac{AD}{DB} = \frac{AE}{EC}$

36. (i) Probability that the selected family has two or three children =  $19+7 = 26\%$  1

(ii) Probability that the selected family has more than two children =  $7+3 = 10\%$  1

(iii) Probability that the selected family has more than one child =  $19+7+3 = 29\%$  2

OR

(iii) Probability that the selected family has less than three children =  $51+20+19 = 90\%$

37. Coordinates of A = (1, 9), B = (5, 13) 1

Coordinates of C = (9, 13), D = (13, 9) 1

$$\text{Midpoint} = \left( \frac{9+13}{2}, \frac{13+9}{2} \right) = (11, 11)$$

Coordinates of M = (5, 11), N = (9, 11) 2

$$MN = \sqrt{4^2 + 0^2} = 4$$

OR

The point which divides MN in the ratio 1:3 is  $\left( \frac{9+15}{4}, \frac{11+33}{4} \right) = (6, 11)$  2

38. (i)  $\angle ROS = \frac{360}{8} = 45^\circ$  1

(ii) Perimeter of sector OPQ =  $21+21+\frac{1}{8} \times 2 \times \frac{22}{7} \times 21 = 58.5\text{cm}$  1

$$(iii)(A) \text{Area of shaded region PQRS} = \frac{1}{8} \times \pi (R^2 - r^2) = \frac{1}{8} \times 3.14 (21^2 - 10^2)$$

$$= \frac{1}{8} \times 3.14 \times 341 = 133.84\text{cm}^2$$

2

OR

(iii)(B) Area of shaded region ACB

2

i.e. the segment ACB

$$= \frac{1}{4} \pi r^2 - \text{area of } \Delta OAB$$

$$= \frac{1}{4} \times 3.14 \times 100 - \frac{1}{2} \times 10 \times 10 = 78.5 - 50 = 28.5\text{cm}$$

