COMMON PRE-BOARD EXAMINATION

MATHEMATICS-Code No. 041

Class-XII-(2025-26)

SET: 3

Marking Scheme

Time allowed: 3 Hrs. **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and strictly follow them:

a) This question paper contains **38** questions. **All** questions are **compulsory**.
b) This question paper is divided into **five** sections – **A, B, C, D** and **E**.
c) In **Section A**, Questions **1** to **18** are multiple choice questions (MCQs) with only one correct option and questions **19** and **20** are Assertion-Reason based questions of **1** mark each.
d) In **Section B**, Questions **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
e) In **Section C**, Questions **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
f) In **Section D**, Questions **32** to **35** are long answer (LA) type questions, carrying **5** marks each.
g) In **Section E**, Questions **36** to **38** are case based questions, carrying **4** marks each.
h) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
i) Use of calculator is **not** allowed.

Q. No.	Questions	Marks
	SECTION A **This section comprises of multiple-choice questions (MCQs) of 1 mark each.** **Select the correct option (Question 1 - Question 18)**	
1.	The domain of the function $\cos^{-1}(2x - 1)$ is $-1 \leq 2x - 1 \leq 1$ $0 \leq x \leq 1$ **Answer** is (a) [0, 1]	1
2.	If A is matrix of order m × n and B is a matrix such that AB' and B'A are both defined, the order of matrix B is **Answer** is (d) m × n	1
3.	If A is a square matrix of order 3 × 3 such that $\lvert A \rvert = 2$, then the value of $\lvert adj(adjA) \rvert$ is **Answer** is (b) 16	1

4. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is symmetric and Q is skew symmetric matrix, then Q is equal to 1

Answer is (b) $\begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$

5. If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is 1

Answer is (a) 0

6. Value of the determinant $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ is 1

Answer is (b) $\frac{\sqrt{3}}{2}$

7. If $y = \tan^{-1} x$ then the value of $(1 + x^2)y_2$ is equal to 1

Answer is (d) $-2xy_1$

8. The value of k for which function 1

$f(x) = \begin{cases} x^2 , & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at $x = 0$

Answer is (a) 0

9. The bottom of a rectangular swimming tank is 25m by 40m water is pumped into the tank at the rate of 500 cubic meters per minute. The rate at which the level of water in the tank is rising is 1

Answer is (d) $\frac{1}{2}\ m/min$

10. $\int \frac{2^{\frac{1}{x}}}{x^2} dx = k2^{\frac{1}{x}} + C$, then k is 1

Answer is (a) $\frac{-1}{\log 2}$

11. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x}$ dx equals 1

Answer is $\log(10^x + x^{10}) + C$

12. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is 1

Answer is (a) $e^x + e^{-y} = C$

13. The respective values of magnitudes of $\vec{a}$ and $\vec{b}$, if given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $\vec{a} = 3\vec{b}$, are: 1

Answer is (c) 24 and 8

14. If $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{c} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$ then a unit vector normal to the vectors $(\vec{a} + \vec{b})$ and $(\vec{b} - \vec{c})$ is 1

Answer is (a) $\hat{\imath}$

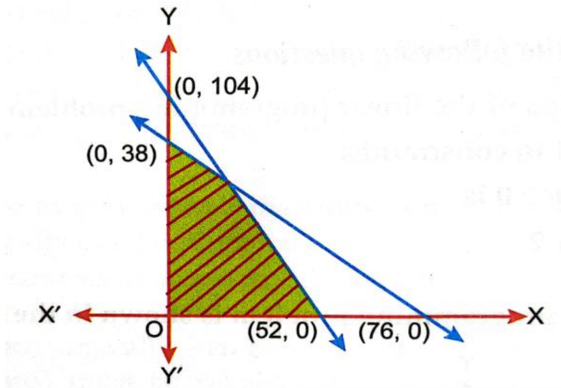
15. If a line makes angles α, β and γ with the axes respectively, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma =$ 1

Answer is (b) -1

16. In an LPP, if the objective function $Z = ax + by$ has the same maximum value at two corner points of the feasible region, then the number of points at which Z_{max} occurs is 1

Answer is (d) infinite

17. The feasible region of a linear programming problem is shown in the figure below: Then, the constraints of the LPP are $x \geq 0, y \geq 0$ and 1



Answer is (b) $2x + y \leq 104$ and $x + 2y \leq 76$

18. For any two events A and B, if $P(\bar{A}) = \frac{1}{2}$, $P(\bar{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{\bar{A}}{\bar{B}}\right)$ is 1

Answer is (c) $\frac{5}{8}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based question carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true, but (R) is false.
(d) (A) is false, but (R) is true.

19. **Assertion (A):** Principal value of $\tan^{-1}(-\sqrt{3})$ is $\frac{-\pi}{3}$. 1

Reason (R): $\tan^{-1} x$: R→$(\frac{-\pi}{2}, \frac{\pi}{2})$ so for any $x \epsilon R, \tan^{-1}(x)$ represents an angle in $(\frac{-\pi}{2}, \frac{\pi}{2})$

Answer is (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

20. **Assertion (A):** Vector equation of the line passing through the points (1, 2, 3) and (5, -4, -7) is $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \boldsymbol{\lambda}(5\hat{\imath} - 4\hat{\jmath} - 7\hat{k})$ where $\boldsymbol{\lambda}$ is a parameter. 1

Reason (R): Vector equation of straight line passing through two given points with position vector $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \boldsymbol{\lambda}(\vec{b} - \vec{a})$

Answer is (d) (A) is false, but (R) is true.

SECTION B

This section comprises of 5 very short answer type questions (VSA) of 2 marks each.

21. a) Find the value of:

$sin\left[\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

$= sin\left[\frac{-\pi}{3} + \pi - \frac{\pi}{6}\right]$ 1

$= sin\left[\frac{\pi}{2}\right]$ ½

= 1 ½

OR

b) Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$

$= \sin^{-1}\left[-\sin\left(2\pi + \frac{\pi}{8}\right)\right]$ 1

$= \sin^{-1}\left[-\sin\left(\frac{\pi}{8}\right)\right]$ ½

$= -\frac{\pi}{8}$ ½

22. If $x = e^{\frac{x}{y}}$, then find $\frac{dy}{dx}$.

$x = e^{\frac{x}{y}}$

$\Rightarrow logx = \frac{x}{y}$

$\Rightarrow ylogx = x$ ½

Differentiating both sides w.r.to x, we get

$\frac{y}{x} + logx\frac{dy}{dx} = 1$ 1

$\Rightarrow \frac{dy}{dx} = \frac{x-y}{xlogx}$ ½

23. If $x^{30}y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

$x^{30}y^{20} = (x + y)^{50}$

Taking log on both sides,

30 log x + 20 log y = 50 log (x + y) ½

Differentiating with respect to x

$\frac{30}{x} + \frac{20}{y}\frac{dy}{dx} = \frac{50}{x+y}\left(1 + \frac{dy}{dx}\right)$ ½

$$\frac{dy}{dx} = \frac{\frac{20x-30y}{x(x+y)}}{\frac{20x-30y}{y(x+y)}}$$ ½

$$\frac{dy}{dx} = \frac{y}{x}$$ ½

24. a) Find: $\int \frac{(x-3)}{(x-1)^3} e^x \, dx$

$$\int \frac{(x-3)e^x}{(x-1)^3} dx = \int \frac{(x-1-2)e^x}{(x-1)^3} dx$$

$$= \int \left(\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right) e^x \, dx = \int \left(\frac{1}{(x-1)^2} + \frac{d}{dx}\left(\frac{1}{(x-1)^2}\right)\right) e^x \, dx$$ 1

$$= \frac{e^x}{(x-1)^2} + c \qquad (\text{as } \int (f(x) + f'(x))e^x \, dx = e^x f(x) + c)$$ 1

OR

b) Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

Let,

$$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\tan x}} . dx$$

$$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \qquad ...(1)$$ ½

Using property,

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)} + \sqrt{\sin\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)}} dx$$

½

$$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \qquad ...(2)$$

Adding (1) and (2)

½

$$2\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1. \, dx$$

$$\text{I} = \frac{\pi}{12}$$ ½

25. Find the direction cosines of the line whose Cartesian equation is:
5x – 3 = 15y + 7 = 3 – 10z
Writing in standard form,

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Direction ratios are,
$a = \frac{1}{5}, b = \frac{1}{15}, c = \frac{-1}{10}$ ½
Direction cosines are given by,

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \frac{b}{\sqrt{a^2+b^2+c^2}},$$ ½

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$\sqrt{a^2 + b^2 + c^2} = \frac{7}{30}$ ½
Direction cosines are: ½
$\left(\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}\right)$

SECTION C

This section comprises of 6 short answer type questions (SA) of 3 marks each.

26. a) If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, then prove that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

Let x $= \sin A, y = \sin B$

$\Rightarrow \sin B\sqrt{1-\sin^2 A} + \sin A\sqrt{1-\sin^2 B} = 1$ ½

$\Rightarrow \sin B \cos A + \sin A \cos B = 1\ [\because \sin(x+y) = \sin x \cos y + \cos x \sin y]$

$\Rightarrow \sin(A+B) = 1$

½

$\Rightarrow A + B = \sin^{-1}(1)$

$\Rightarrow \sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}\ [\because x = \sin A, y = \sin B]$

Differentiate with respect to x,

$\Rightarrow \frac{d}{dx}\left(\sin^{-1} x\right) + \frac{d}{dx}\left(\sin^{-1} y\right) = \frac{d}{dx}\left(\frac{\pi}{2}\right)$ 1

$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$ 1

OR

b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(\log(xe))^2}$ and hence find its value at $x = e$.

Taking log,

$y \log x = (x - y)\log e$

$y \log x + y = x \quad (\text{since } \log e = 1)$ ½

Differentiating with respect to x

$\log x \frac{dy}{dx} + \frac{y}{x} + \frac{dy}{dx} = 1$ ½

$\Rightarrow \frac{dy}{dx}(1 + \log x) = 1 - \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{x-y}{x(1+\log x)} = \frac{x - \frac{x}{1+\log x}}{x(1+\log x)} = \frac{x(1+\log x) - x}{x(1+\log x)^2} = \frac{x(1+\log x - 1)}{x(\log e + \log x)^2} = \frac{\log x}{(\log(xe))^2}$ 1

$\left.\frac{dy}{dx}\right]_{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4}$ 1

27. i) If $(x) = x + \frac{1}{x}, x \geq 1$, show that *f* is an increasing function.

ii) Find the maximum profit that a company can make, if the profit function is given by, $p(x) = 41 - 72\,x - 18\,x^2$

i)

$f'(x) = \frac{d}{dx}\left(x + \frac{1}{x}\right) = 1 - \frac{1}{x^2}$ ½

$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$ ½

$f'(x) = \frac{x^2 - 1}{x^2} \geq 0$

$f'(x) = \frac{x^2 - 1}{x^2} \geq 0$ ½

Therefore *f(x)* is increasing.

ii)

$P'(x) = \frac{d}{dx}(41 - 72x - 18x^2) = -72 - 36x$

½

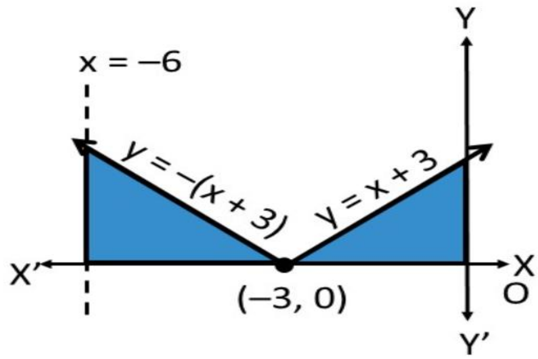
$-72 - 36x = 0 \implies -36x = 72 \implies x = -2$

$P''(x) = \frac{d}{dx}(-72 - 36x) = -36 < 0$ ½

$P(-2) = 41 - 72(-2) - 18(-2)^2 = 41 + 144 - 72 = 113$

Maximum profit $= 113$ ½

28. a) Sketch the graph $y = |x+3|$. Evaluate $\int_{-6}^{0}|x+3|\,dx$.



1

Required area

$= \int_{-6}^{-3} |x+3|\, dx - \int_{-3}^{0} |x+3|\, dx$

$= \int_{-6}^{-3} -(\boldsymbol{x}+\mathbf{3})\, \boldsymbol{dx} + \int_{-3}^{0}(\boldsymbol{x}+\mathbf{3})\, \boldsymbol{dx}$ 1

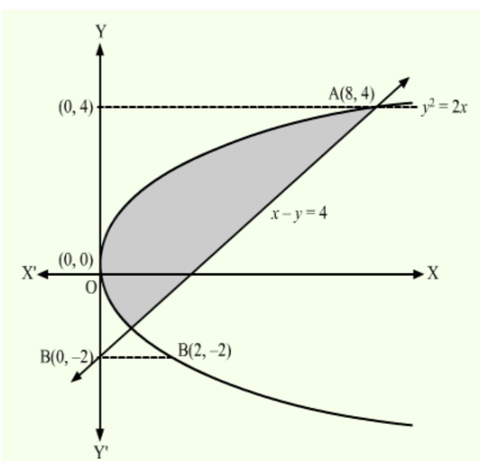
$= \left[-\frac{x^2}{2} - 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$

= 9 square units. 1

OR

b) Find the area of the region bounded by the parabola $y^2 = 2x$ and the line $x - y = 4$.
Solving $y^2 = 2x$ and the line $x - y = 4$, we get
$y = 4, y = -2$

1



1

The points of intersection of the parabola and line are: A (8, 4) and B (2, -2)

Required area $= \int_{-2}^{4}(y+4)\,dy - \int_{-2}^{4}\frac{y^2}{2}\,dy$

$= \left[\frac{(y+4)^2}{2}\right]_{-2}^{4} - \left[\frac{y^3}{6}\right]_{-2}^{4}$

= 30 – 12 1

= 18

29. a) A line passes through (2, -1, 3) and is perpendicular to the lines $\vec{r} = \hat{\imath} + \hat{\jmath} - \hat{k} + \lambda(2\hat{\imath} - 3\hat{\jmath} + \hat{k})$ and $\vec{r} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k} + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$. Obtain its equation in vector and cartesian form.

Let $\vec{b}$ be perpendicular to given lines. Then, $\vec{b}$ is parallel to required line.

$\vec{b} =$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$$= \hat{i}((-3)(2) - 1 \cdot 2) - \hat{j}(2 \cdot 2 - 1 \cdot 1) + \hat{k}(2 \cdot 2 - (-3) \cdot 1)$$

$$= \hat{i}(-6 - 2) - \hat{j}(4 - 1) + \hat{k}(4 + 3)$$

$$= -8\hat{i} - 3\hat{j} + 7\hat{k}.$$ 1

Vector equation of the line is:

$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + t(-8\hat{i} - 3\hat{j} + 7\hat{k})$ 1

Cartesian equation of the line is:

$$\frac{x-2}{-8} = \frac{y+1}{-3} = \frac{z-3}{7}$$ 1

OR

b) Given that $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a triangle such that $\vec{a} = \vec{b} + \vec{c}$. Find p, q, r, s such that area of triangle is $5\sqrt{6}$ where $\vec{a} = p\,\hat{\imath} + q\,\hat{\jmath} + r\hat{k}$, $\vec{b} = s\,\hat{\imath} + 3\,\hat{\jmath} + 4\hat{k}$ and $\vec{c} = 3\,\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\vec{a} = \vec{b} + \vec{c}$

$p = s + 3, q = 4, r = 2$

Area of the triangle = $5\sqrt{6}$

$\frac{1}{2}|\vec{a} \times \vec{b}| = 5\sqrt{6}$ 1

$$\frac{1}{2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ s+3 & 4 & 2 \\ s & 3 & 4 \end{vmatrix} = 5\sqrt{6}$$

Finding determinant and squaring on both sides,

$5s^2 + 30s - 275 = 0$

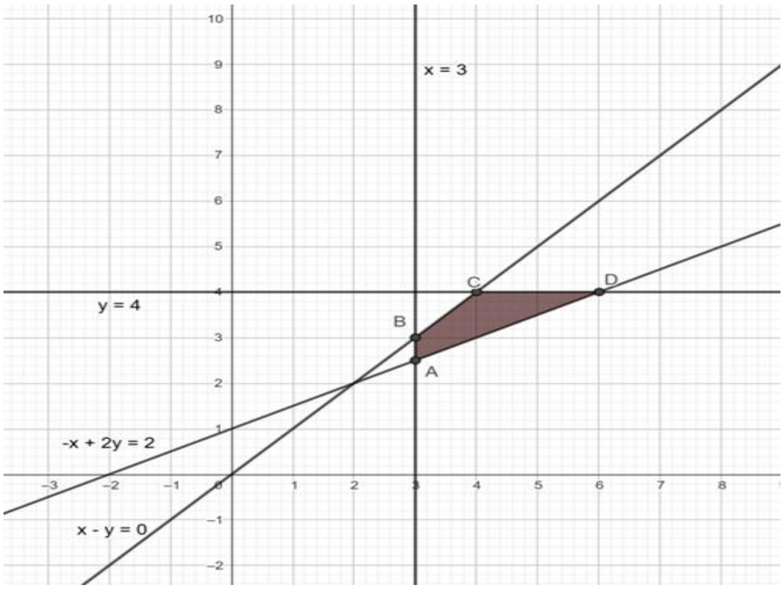
$s^2 + 6s - 55 = 0$ 1

$s = -11 \text{ or } s = 5$ ½

Hence, ½

$p = -8, q = 4, r = 2, s = -11$ or $p = 8, q = 4, r = 2, s = 5$

30. Minimize: $Z = x - 5y$
Subject to constraints: $x - y \geq 0,\ -x + 2y \geq 2,\ x \geq 3,\ y \leq 4,\ y \geq 0$



1½

Corner Point	Value of Z
A (3,2.5)	-9.5
B (3,3)	-12
C (4,4)	-16
D (6,4)	-14

1

Minimum value occurs at x = 4, y = 4
Minimum value is -16 ½

31. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.
   i) Find the probability that he/she reads neither Hindi nor English newspapers.
   ii) If she reads Hindi newspaper, find the probability that she reads English newspaper.

Let H be the event that a student reads the Hindi newspaper, and E be the event that a student reads the English newspaper.
P(H) = 0.6
P(E) = 0.4
P (E ∩ H) = 0.2
i) P (he/she reads neither Hindi nor English newspapers) = P (H’ ∩ B’)
= P (H ∪ E)’ 1
= 1 - P (H ∪ E)
= 1 – (0.6 + 0.4 – 0.2) = 0.2 1

ii) If she reads Hindi newspaper, the probability that she reads English newspaper,
P(E/H) = $\frac{P(E \cap H)}{P(H)} = \frac{0.2}{0.6} = \frac{1}{3}$ 1

SECTION D

This section comprises of 4 long answer type questions (LA) of 5 marks each.

32. If A = $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find $(A)^{-1}$.

Hence, solve the system of linear equations:

$x - 2y = 10$

$2x - y - z = 8$

$-2y + z = 7$

$|A| = 1 \neq 0 \Rightarrow A^{-1}$ exists. 1

$$adjA = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$ 1½

The given system of equations is equivalent to the matrix equation

$A^T X = B$, where $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ½

$\Rightarrow X = (A^T)^{-1} B$

$\Rightarrow X = (A^{-1})^T B$ ½

$$\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\therefore x = 0, y = -5, z = -3$ 1½

33. a) Integrate: $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$

$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \frac{(4x^2+10)}{(x^2+3)(x^2+4)}$ 1

Let,

$\frac{(4x^2+10)}{(x^2+3)(x^2+4)} = \frac{(Ax+B)}{(x^2+3)} + \frac{(Cx+D)}{(x^2+4)}$

$(4x^2 + 10) = (Ax + B)(x^2 + 4) + (Cx + D)(x^2 + 3)$ 1

Equating coefficients and solving, we get,

$A = 0, B = -2, C = 0, D = 6$ 1

$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \frac{(-2)}{(x^2+3)} - \frac{(6)}{(x^2+4)}$

$\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = \int 1 + \frac{(2)}{(x^2+3)} - \frac{(6)}{(x^2+4)} \; dx$ 1

$= x + \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - 3 \; \tan^{-1} \frac{x}{2} + \text{C}$ 1

OR

b) Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2} \; dx$

Let

$x = \tan\theta \;\; \Rightarrow \;\; dx = sec^2\theta \; d\theta$ ½

$I = \int_0^{\frac{\pi}{4}} \frac{\log(1+tan\theta)}{1+tan^2\theta} \cdot sec^2\theta \;\; d\theta$

$I = \int_0^{\frac{\pi}{4}} log\,(1+tan\theta)d\theta \;\; = \;\; \int_0^{\frac{\pi}{4}} log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$ 1

$= \int_0^{\frac{\pi}{4}} \log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right] d\theta$ 1

$= \int_0^{\frac{\pi}{4}} \log\left[\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta}\right] d\theta$

$= \int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan\theta}\right] d\theta$ 1

$= \int_0^{\frac{\pi}{4}} \log 2 \;\; d\theta \;\; - \int_0^{\frac{\pi}{4}} \log[1+tan\theta] \;\; d\theta$

$= \log 2 \times x]_0^{\frac{\pi}{4}} - I$ 1

$\Rightarrow \;\; 2I = \frac{\pi}{4} \log 2$

$\Rightarrow \;\; I = \frac{\pi}{8} \log 2$ ½

34. a) Solve the differential equation:

$(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$, where $x \in (-\infty, -1) \cup (1, \infty)$

The differential equation is:

$(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$

$\frac{dy}{dx} + \frac{2xy}{x^2-1} = \frac{2}{(x^2-1)^2}$ 1

This is of the form $\frac{dy}{dx} + Py = Q$, where P = $\frac{2x}{x^2-1}$ and Q = $\frac{2}{(x^2-1)^2}$ 1

$\therefore IF = e^{\int Pdx} = x^2 - 1$

Solution: 1

$y(x^2-1) = \int \frac{2}{(x^2-1)^2}(x^2-1)dx + C$ 1

$y(x^2-1) = \log\left|\frac{x-1}{x+1}\right| + C$ 1

OR

b) Find the particular solution satisfying the given condition:

$2xy + y^2 - 2x^2 \frac{dy}{dx} = 0; y = 2\ when\ x = 1$

$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$ 1

This is a homogeneous function

Substitute $y = vx$

$v + x\frac{dv}{dx} = v + \frac{v^2}{2}$ 1

$2\int \frac{dv}{v^2} = \int \frac{dx}{x}$ 1

$\frac{-2}{v} = \log|x| + C$

$\frac{-2x}{y} = \log|x| + C$ 1

$y = 2\ when\ x = 1$

$\frac{-2}{2} = \log|1| + C$

C = -1

Particular solution is: 1

$y = \frac{2x}{1 - \log|x|}$

35. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point P (5, 4, 2) to the line $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + \hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$. Also find the image of P in this line.

Let Q(x, y, z) be the foot of perpendicular from P to the given line.

Q lies on the line

(x, y, z) = $(2\lambda - 1, 3\lambda + 3, -\lambda + 1)$ 1

Direction ratios of

$\overrightarrow{PQ}$: $(2\lambda - 6, 3\lambda - 1, -\lambda - 1)$ 1

$\overrightarrow{PQ}$ is perpendicular to $\vec{r}$,

$2(2\lambda - 6) + 3(3\lambda - 1) - 1(-\lambda - 1) = 0$ 1

$\lambda = 1$

Foot of perpendicular from P to the given line is (1, 6, 0)

Length of the perpendicular = $\sqrt{(1-5)^2 + (6-4)^2 + (0-2)^2} = 2\sqrt{6}$ 1

Let R be the image of P.

Then Q is the mid-point of PR.

R is (-3, 8, -2) 1

SECTION E

This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each.

36. **Case Study -1**

A city's traffic management department is planning to optimize traffic flow by analyzing the connectivity between various traffic signals. The city has five major spots labelled $A, B, C, D\ and\ E$.

The department has collected the following data regarding one-way traffic flow between spots:

1. Traffic flows from A to B, A to C and A to D.
2. Traffic flows from B to C and B to E.
3. Traffic flows from C to E.
4. Traffic flows from D to E and D to C.

The department wants to represent and analyze this data using relations and functions. Use the given data to answer the following questions:

i) Is the traffic flow reflexive? Justify. 1

Traffic flow is not reflexive as $(A, A) \notin R$ (or no major spot is connected with itself)

ii) Is the traffic flow transitive? Justify. 1

Traffic flow is not transitive as $(A, B) \in R$ and $(B, E) \in R$, but $(A, E) \notin$ R

iii) a) Represent the relation describing the traffic flow as a set of ordered pairs. Also state the domain and range of the relation.

$R=\{(A,B),(A,C),(A,D),(B,C),(B,E),(C,E),(D,E),(D,C)\}$ 1

Domain = $\{A, B, C, D\}$ ½

Range = $\{B, C, D, E\}$ ½

OR

iii) b) Does the traffic flow represent a function? Justify your answer.

No, the traffic flow doesn't represent a function as A has three images. 1+1

37. **Case Study II:**

Read the following passage and answer the following questions.
Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore.

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2

i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

$V = \pi r^2 h$

Given,

$2\pi rh + \pi r^2 = 75\,\pi$

$h = \frac{75 - r^2}{2r}$ ½

$V = \frac{\pi}{2}(75r - r^3)$ ½

ii) Find the rate of change of V with respect to r.

$\frac{dV}{dr} = \frac{\pi}{2}(75 - 3r^2)$ 1

iii) a) Find the radius of cylinder when its volume is maximum

If $\frac{dV}{dr} = \frac{\pi}{2}(75 - 3r^2) = 0$ ½

$r = 5$ ½

$\frac{d^2V}{dr^2} = -3\pi r < 0$ ½

$\therefore V$ is maximum when $r = 5$ ½

OR

iii) b) For maximum volume, check if $h > r$.

V is maximum when $r = 5$ 1

$h = \frac{75-25}{10} = 5\,cm.$ ½

$\therefore h > r$ is false ½

38. **Case Study III:**

There are two shops in a market A and B. In A there are 30 tin pure Mustard oil, 40 tin adulterated mustard oil, while in B there are 50 tin pure Mustard oil and 60 tin adulterated oil. Babu wants to purchase one tin oil from any shop selecting at random.

i) Find the total probability of purchasing adulterated mustard oil if shop and tin of oil are selected at random.

Let E_1 be the event of choosing store A, E_2 the event of choosing store B and

C be the event of purchasing adulterated mustard oil.

P (E_1) = P (E_2) = $\frac{1}{2}$

P (C/E_1) = $\frac{4}{7}$ ½

P (C/E_2) = $\frac{6}{11}$ ½

Total probability of purchasing adulterated mustard oil = $\frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{6}{11} = \frac{43}{77}$ 1

ii) Babu wants to know quality of mustard oil. Before purchasing he selected first a shop at random and then selected a tin of mustard oil at random. If the tin selected at random has adulterated oil, then find the probability that the selected tin is from shop A.

$$P(E_1/C) = \frac{P(E_1)P(C/E_1)}{P(E_1)P(C/E_1)+P(E_2)P(C/E_2)}$$ 1

$$= \frac{\frac{1}{2}\times\frac{4}{7}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{6}{11}}$$ ½

$$= \frac{22}{43}$$ ½