COMMON PRE-BOARD EXAMINATION

APPLIED MATHEMATICS-Code No. 241

Class-XII-(2025-26)

SET: 1

Time allowed: 3 Hrs. **Maximum Marks: 80**

Q. No.	Questions	Marks
	For **Section-A**, only final correct answers to be considered, no part marks to be given	
1.	B 3 : 2	1
2.	A 2 and 3	1
3.	D $\frac{1}{2}(e-1)$	1
4.	B Singular Matrix	1
5.	B 5	1
6.	D 2(x + 3)	1
7.	B 8	1
8.	C 7	1
9.	D Stratified random sampling	1
10.	B 5%	1
11.	A 2	1
12.	A $x+2y \leq 5; x+y \leq 4$	1
13.	D ₹ 52500	1
14.	B 84.06m	1
15.	A skew symmetric	1
16.	A $\frac{8k^2}{9}$	1
17.	A 1 at each end	1
18.	C The outcomes must be dependent on each other	1
19	D A is false, but R is true	1
20.	B Both (A) and (R) are true and (R) is not the correct explanation of (A).	1
	SECTION-B	
21.	Let n be the number of trails , p be the probability of success in each trail and q = 1 – p. So mean – variance = np – npq = np(1 – q) $\Rightarrow$ mean – variance = np^2 > 0 as n ∈ N and p > 0 $\Rightarrow$ mean > variance **OR** n = 10, p = $\frac{5}{6}$ ∴ q = 1 – $\frac{5}{6}$ = $\frac{1}{6}$ Required probability = P(X < 2) = P(X =0) + P(X = 1) $= 10_{C_0}\left(\frac{5}{6}\right)^{10}\left(\frac{1}{6}\right)^{0} + 10_{C_1}\left(\frac{5}{6}\right)^{9}\left(\frac{1}{6}\right)^{1}$	1 ½ ½ ½ 1

	$= 3 \times \left(\frac{5}{6}\right)^{10}$	1/2
22.	Mean = np = = 0.3 P(X = 0) = $\frac{(0.3)^0 e^{-0.3}}{0!} = 0.7408$ = 74.08% P(X = 1) = $\frac{(0.3)^1 e^{-0.3}}{1!} = 0.2222$ = 22.22%	½ ½ 1
23.	We have AM ≥ GM $\implies \frac{a+b}{2} \geq \sqrt{ab}$, given ab = 1 $\implies \frac{a+b}{2} \geq 1 \implies a+b \geq 2 \implies a+b+2 \geq 4$ (1) Now (1 + a) (1 + b) = 1 + a + b + ab = 1 + a + b + 1 = a + b + 2 (2) By (1) and (2) (1 + a) (1 + b) ≥ 4 **OR** Consider $10 \, mod 7 = 3 \, mod 7 \implies 10^2 \equiv 3^2$ mod7 = 2 mod7 $\implies (10^2)^3 \equiv 2^3$ mod7 = 1 mod7 $\because$ [$\boldsymbol{If\ a \equiv b\ modc\ then\ a^k \equiv b^k\ modc}$] $\implies (10^6)^{85} \equiv 1^{85}$ mod7 = 1mod7 $\implies (10^6)^{85}.10^2.10^2.10 \equiv 1.2.2.3$ mod7 = 12 mod7 = 5 mod 7 Hence $10^{515} \equiv 5$ mod7	½ ½ ½ ½ ½ ½ ½ ½
24.	i) Annual depreciation : $D = \frac{C-S}{n}$ Now $D = \frac{800000-80000}{8} = ₹\,90000$ ii) Now value after 5 years = 800000 – 5 x 90000 = ₹ 3,50,000	½ ½ 1
25.	We know that CAGR = $\left[\left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1\right] \times 100$ Here FV = ₹ 5,50,000 , PV = ₹ 3,00,000, n = 3 ∴ CAGR = $\left[\left(\frac{550000}{300000}\right)^{\frac{1}{3}} - 1\right] \times 100 = \left[(1.833)^{\frac{1}{3}} - 1\right] \times 100$ = [1.2239 – 1] x 100 = 22.39 %	½ 1 ½
	SECTION- C	
26.	Cost of new equipment = ₹50000 Amount required in sinking fund = ₹ 50000 i = $\frac{r\%}{m} = \frac{12}{400} = 0.03$ $\implies 50{,}000 = P \times \frac{(1+0.03)^5-1}{0.03}$ $\implies 50{,}000 = P \times \frac{1.8061-1}{0.03}$ $\implies P = \frac{50000 \times 0.03}{0.8061} = 1860.81$ So, the amount to be deposited quarterly = ₹ 1860.81	½ 1 ½ ½ ½

	OR Let P be the amount financed, so total price of car is (P + 150000). Here EMI = ₹ 25448 , n = 2 x 12 = 24 , i = $\frac{20}{1200} = \frac{1}{60}$ We know that EMI = $\frac{P\,i}{1-(1+i)^{-n}} \implies 25448 = \frac{P\left(\frac{1}{60}\right)}{1-(1+\left(\frac{1}{60}\right))^{-24}}$ $\implies 25448 = \frac{P\left(\frac{1}{60}\right)}{1-\left(\frac{61}{60}\right)^{-24}} \implies 25448 = \frac{P\left(\frac{1}{60}\right)}{1-0.6725} \implies$ 25448 x 0.3275 x 60 = P $\implies$ P = 1,526,880 x 0.3275 = ₹ 500053.20 Hence amount financed = ₹ 500053.20 Actual price of the car = P + 1,50,000 = ₹ (500053.20 + 1,50,000) = ₹ 650053.20	½ 1 ½ ½ ½
27.	Let x be the number of units of Product A produced per day and y be the number of units of Product B produced per day The objective is to **maximize the profit**, which is given by: $Z = 300\,x + 400\,y$ subject to the following constraints: $x + 2y \le 100$ $2x + y \le 80$ $4x \ge y$ $y \le 2x$ $x \ge 0, y \ge 0$	½ ½ 1 1
28.	Let pipe A fill the tank alone in x hours, then B fills in $\frac{x}{2}$ hours and C in $\frac{x}{4}$ hours Now according to the question $\frac{1}{x} + \frac{1}{\frac{x}{2}} + \frac{1}{\frac{x}{4}} = \frac{1}{5} \implies \frac{1}{x} + \frac{2}{x} + \frac{4}{x} = \frac{1}{5}$ $\implies \frac{7}{x} = \frac{1}{5} \implies$ x = 35 hours. So pipe A , B and C takes 35 hours, $17\frac{1}{2}$ hours and $8\frac{3}{4}$ hours respectively.	½ 1 ½ ½ + ½
29.	Given integral I = $\int \frac{2x-1}{(x-1)(x+2)(x-3)}\,dx$ We break the integrand into partial fractions	

$$\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3} \qquad (1)$$ ½

$2x - 1 = A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)$

put x = 1

$2(1) - 1 = A(1+2)(1-3) \Longrightarrow 1 = A(3)(-2) \Longrightarrow A = -\frac{1}{6}$

put x = - 2

$2(-2) - 1 = B(-2-1)(-2-3) \Longrightarrow -5 = B(-3)(-5)$

$\Longrightarrow -5 = 15B \Longrightarrow B = -\frac{5}{15} = -\frac{1}{3}$

put x = 3

$2(3) - 1 = C(3-1)(3+2) \Longrightarrow 5 = C(2)(5) \Longrightarrow 5 = 10C$

$\Longrightarrow C = \frac{5}{10} = \frac{1}{2}$

$\Longrightarrow A = -\frac{1}{6} \quad B = -\frac{1}{3} \quad C = \frac{1}{2}$ ½

Using these values in (1) we get

$$\mathbf{I} = \int \left(\frac{-\frac{1}{6}}{x-1} + \frac{-\frac{1}{3}}{x+2} + \frac{\frac{1}{2}}{x-3} \right) dx$$ ½

$$= -\frac{1}{6}\int \frac{1}{x-1}\,dx - \frac{1}{3}\int \frac{1}{x+2}\,dx + \frac{1}{2}\int \frac{1}{x-3}\,dx$$ 1½

$$\mathbf{I} = -\frac{1}{6}\log|x-1| - \frac{1}{3}\log|x+2| + \frac{1}{2}\log|x-3| + K$$

OR

The given differential equation can be solved using variable separable method

$x(1+y^2)dx - y(1+x^2)dy = 0$

$x(1+y^2)dx = y(1+x^2)dy$

$\frac{x}{1+x^2}dx = \frac{y}{1+y^2}dy$ ½

Integrating both sides

$\int \frac{x}{1+x^2}dx = \int \frac{y}{1+y^2}dy$ ½

let $u = 1+x^2$ & $v = 1+y^2$

so $du = 2xdx \quad dv = 2ydy$.

$\frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\int \frac{1}{v}dv$ ½

$\ln|1+x^2| = \ln|1+y^2| + 2C \Rightarrow \ln(1+x^2) = \ln(1+y^2) + C_1$

$1+x^2 = e^{\ln(1+y^2)+C_1} = e^{\ln(1+y^2)}e^{C_1}$

$1+x^2 = (1+y^2)e^{C_1} \Rightarrow 1+x^2 = C_2(1+y^2)$ ½

	Now using the initial conditions y = 1 when x = 0, we get $1 + (1)^2 = C_2(1 + (0)^2) \implies C_2 = 2$ $1 + x^2 = 2(1 + y^2) \implies y^2 = \frac{x^2 - 1}{2}$	½ ½
30.	Here $\bar{x} = (47 + 41 + 40 + 44 + 43) / 5 = 43$ $\sum(x - \bar{x})^2 = 16 + 1 + 9 + 1 + 0 = 27$ $s^2 = \frac{1}{n-1}\sum(x - \bar{x})^2 \implies s^2 = \frac{27}{4} = 6.75 \implies s = 2.598$ Now we set up the null and alternate hypothesis Null hypothesis $H_0: \mu = 45$ cm Alternate hypothesis $H_1: \mu \neq 45$ cm From above $\bar{x} =$ 43cm and s = 2.598 cm at $\alpha = 0.05$ and df = 4 $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{43 - 45}{\frac{2.598}{\sqrt{5}}} = \frac{-2 \times 2.236}{2.598} = -1.721$ $\lvert tcal \rvert = 1.721 < tcritical = 2.132$ at $\alpha = 0.05$ and df = 4 ∴ null hypothesis is accepted, there is no significant difference between the sample mean and the population mean, hence the claim of the senior meteorologist is correct.	½ ½ ½ ½ ½ ½
31.	Let's speed of the motor boat and the water current be 36x and 5x respectively Then speed downstream is equal to 36x + 5x = 41x and speed up stream is equal to 36x – 5x = 31 x let the distance |AB| be **d** kilometres. Now according to the question $\frac{d}{41x} = 5\frac{10}{60} \Rightarrow d = \frac{31}{6} \times 41x$ Now time taken for return journey $= \frac{d}{31x} = \frac{31}{6 \times 31x} \times 41x = \frac{41}{6}$ hours = 6hrs 50 mins	½ ½ ½ 1 ½
	SECTION- D	
32.	Taking 2014 as the base year and all y values in lakhs	3

Year	Y	X	X^2	XY	Y = a + bx
2010	65	-4	16	-260	66.90
2012	68	-2	4	-136	68.40
2013	70	-1	1	-70	69.15
2014	72	0	0	0	69.90
2015	75	1	1	75	70.65
2016	67	2	4	134	71.40
2019	73	5	25	365	73.65
n = 7	$\sum Y$ =**490**	$\sum x$ =**1**	$\sum X^2$ =**51**	$\sum XY$ =**108**	

Correct table 3 marks

To find the trend line y = a+ b , we use normal equations as $\sum x \neq 0$

Using

$\sum y$ = na + b $\sum x$ and $\sum xy = a\sum x + b\sum x^2$ ½

We get

490 = 7a + b and 108 = a + 51 b

Solving both we get a = 69.9 and b = 0.75 ½ + ½

The trend line is y = 69.9 + 0.75 x ½

OR

Year	Sales	Four yearly total	Four yearly average	Centred moving average
2015	108			
2016	112			
		450	112.50	
2017	110			116.500
		482	120.50	
2018	120			121.500
		490	122.50	
2019	140			121.250
		480	120.00	
2020	120			121.875
		495	123.75	
2021	100			
2022	135			

$1\frac{1}{2}$ mark $1\frac{1}{2}$ mark 2 mark

Deduct half mark if all entries are correct but table formatting is wrong.

33. Since market equilibrium is prevailing so D(x) = S(x)

$\Rightarrow 16 - \frac{x^2}{100} = \frac{x^2}{400} + 6 \Rightarrow \frac{5x^2}{400} = 10 \Rightarrow x = \pm 20\sqrt{2}$

but $x \neq -20\sqrt{2}$ so $x = 20\sqrt{2}$

When $x_0 = 20\sqrt{2}$ *then* $p_0 = 8 \Rightarrow x_0p_0 = 160\sqrt{2}$ 1

i) **C.S.** $= \int_0^{x_0} \boldsymbol{p\,dx} - \boldsymbol{x_0p_0}$

$= \int_0^{20\sqrt{2}} [\,16 - \frac{x^2}{100}\,]\,dx - 160\sqrt{2}$ 1

$= \left[16x - \frac{x^3}{300}\right]_0^{20\sqrt{2}} - 160\sqrt{2} = 320\sqrt{2} - \frac{160\sqrt{2}}{3} - 160\sqrt{2} = \frac{320\sqrt{2}}{3} = ₹\ 150.85$ ½ + ½

ii) **P.S.** $= \boldsymbol{x_0p_0} - \int_0^{x_0} \boldsymbol{p\,dx}$

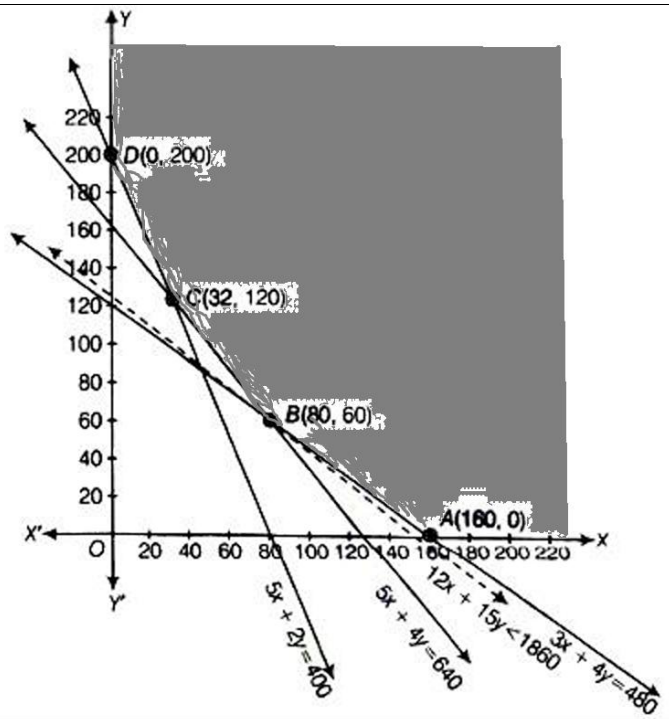
$= 160\sqrt{2} - \int_0^{20\sqrt{2}} [\,\frac{x^2}{400} + 6\,]\,dx$ 1

$= 160\sqrt{2} - \left[\left(\frac{x^3}{1200} + 6x\right)\right]_0^{20\sqrt{2}}$ ½

$= 160\sqrt{2} - \frac{40\sqrt{2}}{3} - 120\sqrt{2} = \frac{80\sqrt{2}}{3} = ₹\ 37.71$ ½

34.	Given that: $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$	
	Now, $A^2 = A.A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$	1
	$\therefore A^2 + xI_2 = yA$	
	$\begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = y\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$	
	$\begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$	1
	Comparing the corresponding entries of equal matrices, we have	
	$\Rightarrow y = 8$ and $16 + x = 3y$	
	$\therefore x = 3 \times 8 - 16 = 8$	
	Hence, the required values of x is 8 and y is 8.	1
	Now $= A^2 + xI = yA \quad \Rightarrow A^2 + 8I = 8A$	
	$\Rightarrow A^{-1}A^2 + 8A^{-1}I = 8A^{-1}A \quad \Rightarrow A + 8A^{-1} = 8I$	1
	$\Rightarrow A^{-1} = \frac{1}{8}(8I - A) = \frac{1}{8}\left(\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}\right) = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$	1
	OR	
	Let x, y and z be the hundreds, tens and unit place digit of the three digit number.	
	As per the conditions given	
	Sum of all digits is 11 ;	
	$x + y + z = 11 \quad (1)$	
	Difference between reversed number and original ;	
	$100z + 10y + x - 5(100x + 10y + z) = 46$	
	$499x + 40y - 95z = 46 \quad (2)$	
	According to third condition	
	$x + 2y - z = 0 \quad (3)$	1
	The matrices representation of above system of equations is	
	$\begin{bmatrix} 1 & 1 & 1 \\ 499 & 40 & -95 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -46 \\ 0 \end{bmatrix}$	½
	Let this be expressed as AX = B, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 499 & 40 & -95 \\ 1 & 2 & 1 \end{bmatrix}$,	
	$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -46 \\ 0 \end{bmatrix}$.	
	As $AX = B \Rightarrow A^{-1}AX = A^{-1}B \Rightarrow X = A^{-1}B$	½
	As $\lvert A \rvert = 1512 \neq 0$ so A^{-1} *exists*	
	adj $A = \begin{bmatrix} 150 & 3 & -135 \\ 404 & -2 & 594 \\ 958 & -1 & -459 \end{bmatrix}$	1
	$\Rightarrow A^{-1} = \frac{1}{\lvert A \rvert} adj.A = \frac{1}{1512}\begin{bmatrix} 150 & 3 & -135 \\ 404 & -2 & 594 \\ 958 & -1 & -459 \end{bmatrix}$	½

	Now $X = A^{-1}B = \frac{1}{1512}\begin{bmatrix}150 & 3 & -135\\ 404 & -2 & 594\\ 958 & -1 & -459\end{bmatrix}\begin{bmatrix}11\\ -46\\ 0\end{bmatrix} = \frac{1}{1512}\begin{bmatrix}1512\\ 4536\\ 10584\end{bmatrix}$ $\boldsymbol{X} = \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 3\\ 7\end{bmatrix} \Longrightarrow$ x = 1 , y = 3 and z = 7 Hence the required three digit number is 137.	1 ½
35.	Face value (F) = ₹50000, periods of interest (n) = 10 x 2 = 20, yield rate (i) = 5/200 = 0.025 Coupon payment = 6 x 50000 ÷ 200 = ₹ 1500 Now redemption price = face value as bond is redeemable at par. Purchase price of bond (P) = $R\left[\frac{1-(1+\mathrm{i})^{-\mathrm{n}}}{\mathrm{i}}\right] + \mathrm{C}(1+\mathrm{i})^{-\mathrm{n}} = 1500\left[\frac{1-(1+0.025)^{-20}}{0.025}\right] + 50000(1+0.025)^{-20}$ $= 1500\left[\frac{1-0.61027}{0.025}\right] + 50000 \text{ x } 0.61027$ = 60000 x 0.38973 + 30513.50 Purchase price of the bond = 23383.80 + 30513.50 = ₹ 53897.30	1 ½ 1 1 1 ½
36.	**i)** P(x) = R(x) – C(x) $\Longrightarrow$ P(x) = xp – C(x) $\Longrightarrow$ P(x) = $x\left(\frac{50}{\sqrt{x}}\right) - (0.5x + 100) = 50\sqrt{x} - \frac{1}{2}x - 100$ **ii)** P'(x) = $\frac{50}{2\sqrt{x}} - \frac{1}{2} = \frac{1}{2}\left(\frac{50}{\sqrt{x}} - 1\right) = \frac{1}{2}(p-1)$ **iii)** For strictly increasing ; P'(x) > 0 $\Longrightarrow \frac{1}{2}\left(\frac{50}{\sqrt{x}} - 1\right) > 0 \Longrightarrow 50 - \sqrt{x} > 0 \Longrightarrow \sqrt{x} < 50 \Longrightarrow x < 2500$ Hence P(x) is strictly increasing in (800, 2500) **OR** For strictly decreasing ; P'(x) < 0 $\Longrightarrow \frac{1}{2}\left(\frac{50}{\sqrt{x}} - 1\right) < 0 \Longrightarrow 50 - \sqrt{x} < 0 \Longrightarrow \sqrt{x} > 50 \Longrightarrow x > 2500$ Hence P(x) is strictly decreasing in (2500, 5000)	1 1 1 + ½ ½ 1 + ½ ½
37.	**i)** Let factory-I run for x days and factory-II run for y days. Total cost (in ₹) = 12000x + 15000y To minimize Z = 12000x + 15000y Subject to the constraints : 50x + 40y ≥ 6400 or 5x + 4y ≥ 640 (1) 50x + 20y ≥ 4000 0r 5x + 2y ≥ 400 (2) 30x + 40y ≥ 4800 or 3x + 4y ≥ 480 (3) x ≥ 0 and y ≥ 0 Now, considering the inequations as equations, we get 5x + 4y = 640 ..... (i) 5x + 2y = 400 ..... (ii) 3x + 4y = 480 ..... (iii) Graphing these lines on coordinate axis system we see that feasible region is unbounded.	½ 1½



1

Corner Point	Z = 12000x + 15000y
A(10,0)	ZA = 12000 × 160 + 0 = 1920000
B(0,8)	ZB = 12000 × 80 + 15000 × 60 = 186000 (minium)
C(2,4)	ZC = 12000 × 32 + 15000 × 120 = 2184000
D(0.200)	ZD = 0 + 15000 × 200 = 3000000

½

We see that minimum value of Z is 1860000, occur at the point B(80, 60). But we can't say that it is a minimum value of Z as region is unbounded. Therefore, we have to draw the graph of the inequality 12000z + 15000 < 1860000 or 12x + 15y < 1860
From the figure, we see that the open half plane represented by 12x + 15y < 1860 has no point in common with feasible region.
Factory (I) should run for 80 days and factory (II) should run for 60 days to get a minimum cost of ₹ 1860000.

½

38.

Here, $\mu = 75, \sigma = 8, n = 500$

(i) For $X = 75, Z = \frac{X-\mu}{\sigma} = \frac{75-75}{8} = 0$

½

$P(X < 75) = P(Z < 0) = 0.5$

½

50 % of students scored below 75 marks.

(ii) For $X = 82, Z = \frac{82-75}{8} = 0.875$

½

$P((X > 82) = P(Z > 0.875) = 1 - P(Z < 0.875) = 1 - 0.8092 = 0.1908$

½

∴ Required number of students $= 0.1908 \times 500 = 95.4 \approx 95$

	(iii) (a) For $X = 67, Z = \frac{67-75}{8} = -1$ For $X = 83, Z = \frac{83-75}{8} = 1$ $P(67 < X < 83) = P(-1 < Z < 1)$ $= P(Z < 1) - P(Z < -1)$ $= 0.8413 - 0.1587 = 0.6826$ ∴ Required number of students $= 0.6826 \times 500 = 341.3 \approx 341$ **OR** (iii) (b) Top 10% corresponds to the 90th percentile. $\Longrightarrow Z = \frac{X-\mu}{\sigma} = 1.28$ $\Longrightarrow \frac{X-75}{8} = 1.28$ $\Longrightarrow X = 85.24 \approx 85$ ∴ The minimum score required to qualify for the scholarship is 85 marks.	½ **1** ½ ½ **1** **1**