COMMON PRE-BOARD EXAMINATION 2025-26

MATHEMATICS -CODE NO. 041

CLASS-XII-(2025-26)

SET: 3 ANSWER KEY

SECTION-A **[1 × 20 = 20]**

(This section comprises of multiple-choice questions (MCQs) of 1 mark each.)

Select the correct option (Question 1 - Question 20)

1.	(b) a^9	11.	(a) $3p = q$	
2.	(d) $\frac{\pi}{6}$	12.	(d) 300	
3.	(d) -3	13.	(b) $\frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right)+C$	
4.	(b) $12, -2$	14.	(b) $\frac{1}{\pi}$ unit	
5.	(a) $\frac{5}{8}$	15.	(b) -1	
6.	d) any point on the line segment joining the points$(0,2)$ and $(3,0)$.	16.	(c) $\frac{3}{2}$	
7.	(b) $\frac{x^{2x}}{2}+C$	17.	(c) 0	
8.	(d) $12\sqrt{3}$	18.	(c) 4	
9.	(c) 25	19.	(c) A is true but R is false	
10.	(a) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	20.	(c)A is true but R is false	

SECTION B **[2 × 5 = 10]**

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21.	$\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = 1$ $\lvert\sqrt{2}\vec{a}-\vec{b}\rvert = 1$ $\Rightarrow \lvert\sqrt{2}\vec{a}-\vec{b}\rvert^2 = 1$ $\Rightarrow (\sqrt{2}\vec{a}-\vec{b}).(\sqrt{2}\vec{a}-\vec{b}) = 1$ $\Rightarrow 2\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 - 2\sqrt{2}\vec{a}.\vec{b} = 1$	0.5 0.5

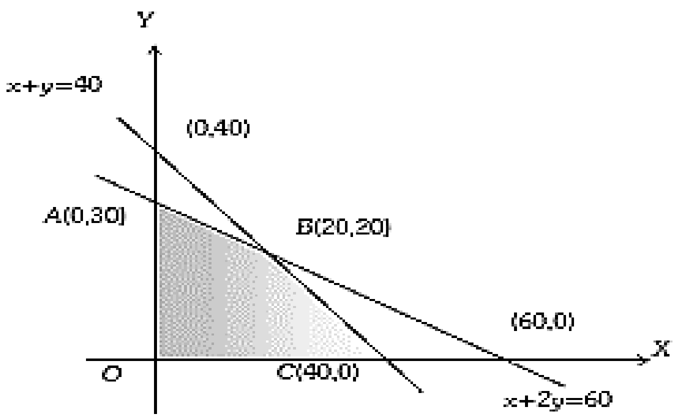
	$\Rightarrow 3 - 2\sqrt{2}\lvert\vec{a}\rvert\lvert\vec{b}\rvert cos\theta = 1$ $\Rightarrow cos\theta = {}^{1}/_{\sqrt{2}}$ $\Rightarrow \theta = \frac{\pi}{4}$	0.5 0.5
22.	$\sqrt{x} + \sqrt{y} = 1$ Differentiating both sides w.r.t x $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$ $\frac{dy}{dx}$ at $\left(\frac{1}{9}, \frac{1}{9}\right)$ is -1.	1 0.5 0.5
23.	$x = a\left(cos\theta + logtan\frac{\theta}{2}\right)$ $\qquad$ $y = asin\theta$ Differentiating x w.r.t θ $\qquad$ Differentiating y w.r.t θ $\frac{dx}{d\theta} = a\left(-sin\theta + \frac{1}{tan\frac{\theta}{2}} \times sec^2\frac{\theta}{2} \times \frac{1}{2}\right)$ $\qquad$ $\frac{dy}{d\theta} = acos\theta$ $\frac{dx}{d\theta} = a\left(-sin\theta + \frac{1}{2sin\frac{\theta}{2}cos\frac{\theta}{2}}\right)$ $= a\left(-sin\theta + \frac{1}{sin\theta}\right)$ $= a\left(\frac{-sin^2\theta + 1}{sin\theta}\right)$ $= a\left(\frac{cos^2\theta}{sin\theta}\right)$	1 0.5

	$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} \quad = \frac{a\cos\theta}{a\left(\frac{\cos^2\theta}{\sin\theta}\right)} = \tan\theta$	0.5
24.	$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ $= \tan^{-1}\left[2\sin\left(2\left(\frac{\pi}{6}\right)\right)\right]$ $= \tan^{-1}\left[2\sin\left(\frac{\pi}{3}\right)\right]$ $= \tan^{-1}\left[2\left(\frac{\sqrt{3}}{2}\right)\right]$ $= \tan^{-1}[\sqrt{3}] = \frac{\pi}{3}$ **OR** $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$ $= \frac{\pi}{4} + \pi - \frac{\pi}{3} - \frac{\pi}{6}$ $= \frac{3\pi+12\pi-4\pi-2\pi}{12}$ $= \frac{9\pi}{12} = \frac{3\pi}{4}$	0.5 0.5 0.5 0.5 1 0.5 0.5
25.	$I = \int_{-2}^{2} \frac{x^2}{1+5^x}\,dx$ ……… (1) By applying the property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ $I = \int_{-2}^{2} \frac{(-x)^2}{1+5^{-x}}\,dx$ ……… (2) (1) + (2) $\Longrightarrow 2I = \int_{-2}^{2} \left[\frac{x^2}{1+5^x} + \frac{(-x)^2}{1+5^{-x}}\right] dx$ $= \int_{-2}^{2} \left[\frac{x^2}{1+5^x} + \frac{5^x x^2}{1+5^x}\right] dx = \int_{-2}^{2} x^2\,dx$ $= \left[\frac{x^3}{3}\right]_{-2}^{2} = \frac{16}{3}$ $\therefore I = \frac{8}{3}$	0.5 0.5 0.5 0.5

	OR $\int_0^{\frac{\pi}{4}} \sqrt{1+sin2x}dx$ $= \int_0^{\frac{\pi}{4}} \sqrt{(sinx+cosx)^2}dx$ $= \int_0^{\frac{\pi}{4}} (sinx+cosx)dx$ $= [-cosx+sinx]_0^{\frac{\pi}{4}}$ $= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (-1+0) = 1$	1 0.5 0.5
	SECTION C **[3 × 6 =18]** **(This section comprises of 6 short answer (SA) type questions of 3 marks each.)**	
26.	$y = \frac{4\sin\ \theta}{(2+\cos\ \theta)} - \theta$ $\frac{dy}{d\theta} = \frac{(2+\cos\theta)4\cos\theta - 4\sin\ \theta(-sin\theta)}{(2+\cos\ \theta)^2} - 1$ $\frac{dy}{d\theta} = \frac{8\cos\theta + 4cos^2\theta + 4\sin^2\theta - 4 - 4cos\theta - cos^2\theta}{(2+\cos\theta)^2}$ $= \frac{4cos\theta - cos^2\theta}{(2+\cos\theta)^2} = \frac{cos\theta(4-cos\theta)}{(2+\cos\theta)^2}$ For $\theta \epsilon \left[0, \frac{\pi}{2}\right], cos\theta \geq 0, 4 - cos\theta > 0\ and\ (2+\cos\theta)^2 > 0$ $\Longrightarrow \frac{cos\theta(4-cos\theta)}{(2+\cos\theta)^2} \geq 0$ $\therefore y$ is an increasing function	1.5 0.5 0.5 0.5
27.	$5x - 3 = 15y + 7 = 3 - 10z$ Equation of the line is $\frac{x - \frac{3}{5}}{\frac{1}{5}} = \frac{y + \frac{7}{15}}{\frac{1}{15}} = \frac{z - \frac{3}{10}}{\frac{-1}{10}}$	1

	The dr's are $\langle \frac{1}{5}, \frac{1}{15}, \frac{-1}{10} \rangle$ or $\langle 6,2,-3 \rangle$ The dc's are $\langle \frac{6}{7}, \frac{2}{7}, \frac{-3}{7} \rangle$ The point at which it passes is $\left(\frac{3}{5}, \frac{-7}{15}, \frac{3}{10}\right)$ **OR** The vector equation of the line passing through $(1,2,-4)$is $\vec{r} = \hat{\imath} + 2\hat{\jmath} + 6\hat{k} + \lambda\left(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}\right)$ Cartesian Equation is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-6}{1}$ Equation of the line passing through the points A$(3,3,-5)$ and B$(1,0,-11)$ is $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu\left(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}\right)$ Distance between the parallel lines is $d = \left\|\frac{(\vec{a_2} - \vec{a_1}) \times \vec{b}}{\|\vec{b}\|}\right\| = \left\|\frac{(2\hat{\imath}+\hat{\jmath}-\hat{k})\times(2\hat{\imath}+3\hat{\jmath}+6\hat{k})}{\|2\hat{\imath}+3\hat{\jmath}+6\hat{k}\|}\right\|$ $= \left\|\frac{9\hat{\imath}-14\hat{\jmath}+4\hat{k}}{7}\right\| = \frac{\sqrt{293}}{7}\ units$	0.5 1 0.5 1 0.5 1.5
28.	E_1: 2 balls are white E_2: 3 balls are white E_3: 4 balls are white A : Two balls are white $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ $P\left(A/E_1\right) = \frac{1}{6}, P\left(A/E_2\right) = \frac{1}{2}, P\left(A/E_3\right) = 1$ $P\left(E_3/A\right) = \frac{P(E_3)\,P\left(A/E_3\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_3)P\left(A/E_3\right)}$ $= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1} = \frac{6}{10} = \frac{3}{5}$	 0.5 0.5 2

29.



1.5

Corner points	$Z = 3x + 4y$
$A(0,30)$	120
$B(20,20)$	140
$C(40,0)$	120
$O(0,0)$	0

1

Maximum is at $B(20,20)$ and maximum value is 140. 0.5

30.

$y = \sin(m\sin^{-1}x) \Rightarrow \frac{dy}{dx} = \cos(m\sin^{-1} x) \cdot \frac{m}{\sqrt{1-x^2}}$ 1

$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = m\cos(m\sin^{-1} x)$

Again diff.w.r.t. x, $\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{-2x}{\sqrt{1-x^2}}\right) = -m\sin(m\sin^{-1} x)\frac{m}{\sqrt{1-x^2}}$ 1

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -m^2\sin(m\sin^{-1} x) = -m^2y$ 0.5

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$ 0.5

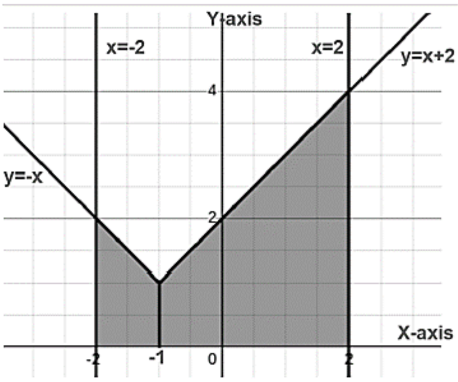
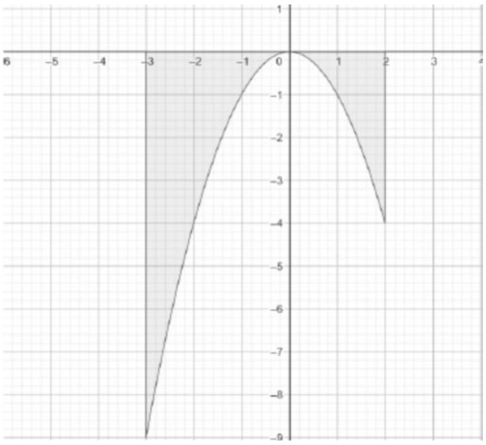
OR

$y = (\log x)^x + x^{\log x} = e^{\log\{(\log x)^x\}} + e^{\log\{x^{\log x}\}}$ 0.5

$= e^{x\log\{(\log x)\}} + e^{\log x \cdot \log x}$ 2

$\frac{dy}{dx} = e^{x\log\{(\log x)\}}\left[x \cdot \frac{1}{\log x} \cdot \frac{1}{x} + \log(\log x) \cdot 1\right] + e^{\log x \cdot \log x}\left[\frac{\log x}{x} + \frac{\log x}{x}\right]$ 0.5

$= (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x}\left[\frac{2\log x}{x}\right]$

Q. No.	Solution	Marks
31.		0.5
	$y = 1 + \|x+1\| = \begin{cases} 1 - x - 1 \ , x < -1 \\ 1 + x + 1, x \geq -1 \end{cases}$	0.5
	$y = \begin{cases} -x \ , x < -1 \\ x + 2, x \geq -1 \end{cases}$	
	RA = Area under $y = x$ +Area under $y = (x+2)$	
	$= \int_{-3}^{-1} -x\, dx + \int_{-1}^{3} x + 2\, dx$	1
	$= -\left(\frac{x^2}{2}\right)_{-3}^{-1} + \left(\frac{x^2}{2} + 2x\right)_{-1}^{3}$	
	$= -\left(\frac{1}{2} - \frac{9}{2}\right) + \left[\frac{9}{2} + 6 - \left(\frac{1}{2} - 2\right)\right]$	1
	= 16 squnits	
	OR	
		1
	RA $= \left\| \int_{-3}^{2} -x^2\, dx \right\|$	1
	$= \left\| -\left(\frac{x^3}{3}\right)_{-3}^{2} \right\| = \left\| \frac{-1}{3}(8 - (-27)) \right\| = \frac{35}{3}$sq units	1
	SECTION D **[5 × 4 = 20]** **(This section comprises of 4 long answer (LA) type questions of 5 marks each)**	

32.	$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$	
	$AB = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$	2
	$\Rightarrow AB = 9I_3$	
	$\Rightarrow A^{-1} = \frac{1}{9}B$	0.5
	For the given system of linear equations	
	$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$	0.5
	$X = A^{-1}C$	
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}BC$	
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$	1
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$	0.5
	$x = 1, y = 2, z = 3$	0.5
33.	Equation of the line is $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$	
	General point on the line is $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$	1
	Distance from the point $A(1,2,3)$ is $3\sqrt{2}$	1
	$\Rightarrow (3\lambda - 2 - 1)^2 + (2\lambda - 1 - 2)^2 + (2\lambda + 3 - 3)^2 = \left(3\sqrt{2}\right)^2$	

	$\Longrightarrow (3\lambda-3)^2+(2\lambda-3)^2+(2\lambda)^2=18$ $\Longrightarrow 17\,\lambda^2-30\lambda=0$ $\Longrightarrow\ \lambda=0\ or\ \lambda=\dfrac{30}{17}$ $\therefore$ Point is $(-2,-1,3)$ or $\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$	1 1 1
34.	$2ye^{\frac{x}{y}}dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$ $\Rightarrow \dfrac{dx}{dy}=\dfrac{2xe^{x/y}-y}{2y\cdot e^{x/y}}$ Let $F(x,y)=\frac{2xe^{x/y}-y}{2y\cdot e^{x/y}}$ $F(\lambda x,\lambda y)=\dfrac{2\lambda xe^{\lambda x/\lambda y}-\lambda y}{2\lambda y\cdot e^{\lambda x/\lambda y}}=\lambda^0\left(\dfrac{2xe^{x/y}-y}{2y\cdot e^{x/y}}\right)=\lambda^0F(x,y)$ $\therefore$ given Diff.Eq. is homogeneous. Let $x=vy\Rightarrow\frac{dx}{dy}=v+x\frac{dv}{dy}$ $\Rightarrow v+x\dfrac{dv}{dy}=\dfrac{2vye^{vy/y}-y}{2y\cdot e^{vy/y}}=\dfrac{2ve^v-1}{2.e^v}$ $\Rightarrow x\dfrac{dv}{dy}=\dfrac{2ve^v-1}{2.e^v}-v=\dfrac{2ve^v-1-2ve^v}{2.e^v}=-\dfrac{1}{2.e^v}$ $2\int e^v dv=-\int\dfrac{dy}{y}\Rightarrow 2e^v+\log\,y\|=C\Rightarrow 2e^{x/y}+\log\|y\|\ =C$ $\because$ y = 1 when x = 0 ⇒ C = 2 $\therefore$ required sol is $\mathbf{2e^{x/y}+logy\ \|=C}$ **OR** Let $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\therefore v+x\dfrac{dv}{dx}=\dfrac{v^2-3xvx}{x^2+xvx}=\dfrac{v^2x^2-3xvx}{x^2+xvx}=\dfrac{v^2-3v}{1+v}$ $\therefore x\dfrac{dv}{dx}=\dfrac{v^2-3v}{1+v}-v=\dfrac{v^2-3v-v-v^2}{1+v}=\dfrac{-4v}{1+v}$ $\therefore\int\left(\dfrac{1+v}{v}\right)dv=-4\int\dfrac{dx}{x}\Rightarrow\int\left(\dfrac{1}{v}\right)dv+\int dv=-4\int\dfrac{dx}{x}$ $\Rightarrow\log\,v+v+4\log\,x=C\Rightarrow\log\dfrac{y}{x}\cdot x^4+\dfrac{y}{x}=C$ $\Rightarrow \mathbf{x\log(x^3y)=Cx}$	0.5 0.5 1.5 1 1 0.5 0.5 2 1 1 0.5

35.	$I = \int_0^{\frac{\pi}{4}} \frac{sinx + cosx}{9 + 16sin2x}\, dx$ $= \int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{25-16(1-sin2x)}\, dx$ $= \int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{25-16(sinx-cosx)^2}\, dx$ Let $sinx - cosx = t \Longrightarrow (cosx + sinx)dx = dt$ When $x = 0, t = -1\ and\ x = \frac{\pi}{4}\ t = 0$ $= \int_{-1}^{0} \frac{t}{25 - 16t^2}\, dt$ $= \frac{1}{16}\int_{-1}^{0} \frac{t}{\frac{25}{16} - t^2}\, dt = \frac{1}{4} \times \frac{1}{10}\left[log\left[\frac{5+4t}{5-4t}\right]\right]_{-1}^{0}$ $= \frac{1}{40}\left[log1 - log\frac{1}{9}\right] = \frac{1}{40}log9 = \frac{1}{20}log3$ OR $\int x^2\tan^{-1} x \cdot dx = \tan^{-1} x\int x^2 \cdot dx - \int \left[\left(\int x^2 \cdot dx\right)\frac{d}{dx}(\tan^{-1} x)\right]dx$ $= \tan^{-1} x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{1+x^2} \cdot dx = \tan^{-1} x \cdot \frac{x^3}{3} - \frac{1}{3}\int \frac{x^3}{1+x^2} \cdot dx$ $= \tan^{-1} x \cdot \frac{x^3}{3} - \frac{1}{3}\int \frac{x^3 + x - x}{1+x^2} \cdot dx$ $= \tan^{-1} x \cdot \frac{x^3}{3} - \frac{1}{3}\int \frac{x(x^2+1) - x}{1+x^2} \cdot dx$ $= \tan^{-1} x \cdot \frac{x^3}{3} - \frac{1}{3}\int x \cdot dx + \frac{1}{6}\int \frac{2x}{1+x^2} \cdot dx$ $= \frac{x^3}{3}\tan^{-1} x - \frac{1}{6}x^2 + \frac{1}{6}\log(x^2+1) + C$	 1 1 0.5 1.5 1 1.5 1.5 1 0.5 0.5
	SECTION- E **[4 × 3= 12]** **(This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (a), (b), (c) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each)**	
36.	a) $P(E/A_1) + P(E/A_2) + P(E/A_3) = \frac{45}{100} + \frac{60}{100} + \frac{35}{100} = \frac{14}{10}$	2

	b) $P(E) = P(A_1)\ P(E/A_1) + P(A_2)P(E/A_2) + \ P(A_3)P(E/A_3)$ $= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100} = \frac{490}{1000} = 49\%$ **OR** $P\left({}^{A_2}/_{E'}\right) = \frac{P(A_2)\ P(E'/A_2)}{P(A_1)P(E'/A_1) + P(A_2)\ P(E'/A_2) + P(A_3)\ P(E'/A_3)}$ $= \frac{\frac{4}{10} \times \frac{40}{100}}{\frac{4}{10} \times \frac{55}{100} + \frac{4}{10} \times \frac{40}{100} + \frac{2}{10} \times \frac{65}{100}} = \frac{16}{51}$	2
37.	a) Area $A = \frac{12x}{5}\sqrt{25 - x^2}$	1
	b) $\frac{dA}{dx} = 0 \Longrightarrow \frac{12}{5} \times \frac{(25-2x^2)}{\sqrt{25-x^2}} = 0 \Longrightarrow x = \frac{5}{\sqrt{2}}$	1
	c) For values of $x < \frac{5}{\sqrt{2}}$, $\frac{dA}{dx} > 0$ and for values of $x > \frac{5}{\sqrt{2}}$, $\frac{dA}{dx} < 0$ Hence, by the first derivative test, local maximum is at the critical point $= \frac{5}{\sqrt{2}}$. For maximum area ,length $= 5\sqrt{2}\, units$ and breadth= $3\sqrt{2}\, units$ **OR** Area $A = \frac{12x}{5}\sqrt{25 - x^2}$ Let $Z = A^2 = \frac{144x^2}{25}(25 - x^2)$ Diff Z w.r.t. x $\frac{dZ}{dx} = \frac{144}{25}(50x - 4x^3)$ $\frac{dZ}{dx} = 0 \Longrightarrow\ x = \frac{5}{\sqrt{2}}$ Diff $again$ w.r.t. x $\frac{d^2Z}{dx^2} = \frac{144}{25}(50 - 12x^2)$	2

	$\left[\frac{d^2Z}{dx^2}\right]_{x=\frac{5}{\sqrt{2}}} < 0$ Hence, by the second derivative test, local maximum value of Z is at the critical point $x = \frac{5}{\sqrt{2}}$ Area A is maximum at the critical point $x = \frac{5}{\sqrt{2}}$. Area A is maximum when length $= 5\sqrt{2}\,units$ and breadth= $3\sqrt{2}\,units$	
38.	a) R_1	1
	b) R_4	1
	c) $\{(1,1),(2,2),(3,3),(2,1),(3,1),(2,3)\}$	2