



# COMMON PRE-BOARD EXAMINATION

MATHEMATICS - Code No. 041

CLASS: XII (2025-26) - SET- 2



Time Allowed: 3 hours

ANSWER KEY

Maximum Marks: 80

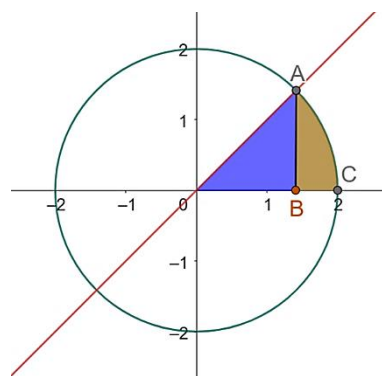
NO	SECTION A	MARKS
----	-----------	-------

1	(b) 3	1	11	(a) $\sqrt{507}$	1
2	(b) 2	1	12	(c) $\sqrt{2}$	1
3	(d) $\left[0, \frac{2}{3}\right]$	1	13	(b) The open half plane not containing origin.	1
4	(a) Skew symmetric matrix	1	14	(b) (0, 8)	1
5	(c) 1/8	1	15	(c) -4	1
6	(c) 12 cm <sup>2</sup> /sec	1	16	(c) not defined	1
7	(c) $-\cos x + C$	1	17	(d) $ A  \in [2, 4]$	1
8	(c) $\sec^2 y \tan y$	1	18	(d) 40	1
9	(a) $\frac{-2}{3}$	1	19	(d) A is false but R is true.	1
10	(b) $\sec x$	1	20	(c) A is true and R is false.	1

SECTION B		
21	$f(0) = 3$ $LHL = \lim_{x \rightarrow 0^-} \frac{kx}{-x} = -k$ $RHL = \lim_{x \rightarrow 0^+} 3 = 3$ Since f is continuous at x = 0, $LHL = RHL - f(0) \Rightarrow -k = 3 \Rightarrow k = -3$	0.5 0.5 0.5 0.5
22	$3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$	1.5

	$\Rightarrow 3 \times \pi/4 + 2 \times \pi/6 + \pi/2$ $\Rightarrow \frac{19\pi}{12}$ <p style="text-align: center;"><b>OR</b></p> $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$ $\Rightarrow \frac{\cos x}{1-\sin x} = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} - 2\sin \frac{x}{2} \cos \frac{x}{2}} = \frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} = \frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}}$ $= \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ $\tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) = \frac{\pi}{4} + \frac{x}{2}$	0.5  1 0.5 0.5
23	$y = 5e^{7x} + 6e^{-7x}$ $\frac{dy}{dx} = 35e^{7x} - 42e^{-7x}$ $\frac{d^2y}{dx^2} = 35 \cdot 7e^{7x} - 42 \cdot (-7)e^{-7x} = 49(5e^{7x} + 6e^{-7x}) = 49y$ $\frac{d^2y}{dx^2} = 49y$	0.5  1 0.5
24	$\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}, \vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}.$ $\vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$ $(\vec{b} + \vec{c}) \cdot \vec{a} = 6 - 2 + 2 = 6,  \vec{a}  = \sqrt{4 + 4 + 2} = 3$ $\text{Projection of } (\vec{b} + \vec{c}) \text{ on } \vec{a} = \frac{(\vec{b} + \vec{c}) \cdot \vec{a}}{ \vec{a} } = \frac{6}{3} = 2$	0.5  0.5 + 0.5  0.5
25	$\int \frac{2^{x+1} - 5^{x-1}}{10^x} dx$ $\frac{2^{x+1}}{10^x} = 2 \cdot 5^{-x}, \frac{5^{x-1}}{10^x} = \frac{1}{5} \cdot 2^{-x}$ $\int \left(2 \cdot 5^{-x} - \frac{1}{5} \cdot 2^{-x}\right) dx = (-2) \cdot \frac{5^{-x}}{\log 5} - (-1) \cdot \frac{1}{5} \cdot \frac{2^{-x}}{\log 2} + C$ $= -\frac{2}{5^x \cdot \log 5} + \frac{1}{5 \cdot 2^x \cdot \log 2} + C$ <p style="text-align: center;"><b>OR</b></p> $y = \sin x, [0, 2\pi]$ $\text{Required Area} = \int_0^\pi \sin x dx + \left  \int_\pi^{2\pi} \sin x dx \right $ $= (-\cos x)_0^\pi +  (-\cos x)_\pi^{2\pi} $ $= -(\cos \pi - \cos 0) +  -(\cos 2\pi - \cos \pi) $ $= -(-1 - 1) +  -(1 + 1)  = 2 + 2 = 4 \text{ sq. unit}$	0.5  1 0.5  0.5 0.5 0.5

SECTION C		
26	$f(x) = \sin^2 x - \cos x, x \in [0, \pi]$ $f'(x) = 2\sin x \cdot \cos x + \sin x$ $f'(x) = 0 \Rightarrow 2\sin x \cdot \cos x + \sin x = 0 \Rightarrow \sin x(2\cos x + 1) = 0$ $\sin x = 0 \Rightarrow x = 0, \pi$ $\cos x = -1/2 \Rightarrow x = 2\pi/3$ $f(0) = -1, f(2\pi/3) = 5/4, f(\pi) = 1$ Absolute maximum = 5/4 at $x = 2\pi/3$ Absolute minimum = -1 at $x = 0$	1  1  0.5  0.5
27	$I = \int \frac{dx}{\sqrt{\sin^3 x \cos(x-a)}} = \int \frac{dx}{\sqrt{\sin^3 x (\cos x \cdot \cos a + \sin x \cdot \sin a)}}$ $\int \frac{dx}{\sin^2 x \sqrt{\cot x \cdot \cos a + \sin a}} = \int \frac{\operatorname{cosec}^2 x \cdot dx}{\sqrt{\cot x \cdot \cos a + \sin a}}$ Put, $t = \cot x \cdot \cos a + \sin a \Rightarrow \frac{dt}{dx} = -\operatorname{cosec}^2 x \cdot \cos a$ $\frac{-dt}{\cos a} = \operatorname{cosec}^2 x dx$ $I = \frac{-1}{\cos a} \int \frac{dt}{\sqrt{t}} = -\frac{1}{\cos a} \times 2\sqrt{t} \Rightarrow I = -\frac{2\sqrt{\cot x \cdot \cos a + \sin a}}{\cos a} + C$ <p style="text-align: center;"><b>OR</b></p> $x^2 + y^2 = 4, x = \sqrt{3}y$ $x^2 + y^2 = 4 \Rightarrow y = \sqrt{4 - x^2}$ $\Rightarrow x = \pm\sqrt{3}$ <p>Required area of the shaded region =</p> $\int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{\sqrt{3}}^2 \sqrt{4 - x^2} dx$ $= \frac{\sqrt{3}}{2} + \left( \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right)_{\sqrt{3}}^2$ $= \frac{\sqrt{3}}{2} + (0 + 2\sin^{-1} 1) - \left( \frac{\sqrt{3}}{2} \times 1 + 2\sin^{-1} \frac{\sqrt{3}}{2} \right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$	0.5  1  0.5  1  1  0.5



28	$x = \cos t(3 - 2\cos^2 t) \Rightarrow x = \cos 3t \Rightarrow \frac{dx}{dt} = -3\sin 3t$ $y = \sin t(3 - 2\sin^2 t) \Rightarrow y = \sin 3t \Rightarrow \frac{dy}{dt} = 3\cos 3t$ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\cos 3t}{-3\sin 3t} = -\cot 3t$ $t = \frac{\pi}{4} \Rightarrow \frac{dy}{dx} = -\cot \frac{3\pi}{4} = -(-1) = 1$ <p style="text-align: center;"><b>OR</b></p> $y = (\sin x)^x + \sin^{-1}\sqrt{x}; u = (\sin x)^x; v = \sin^{-1}\sqrt{x}$ $u = (\sin x)^x \Rightarrow \log u = x \log(\sin x)$ $\frac{du}{dx} = (\sin x)^x (\log(\sin x) + x \cot x)$ $v = \sin^{-1}\sqrt{x} \Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x-x^2}}$ $\frac{dy}{dx} = (\sin x)^x (\log(\sin x) + x \cot x) + \frac{1}{2\sqrt{x-x^2}}$	<p>1</p> <p>1</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>1</p> <p>1</p> <p>0.5</p>
29	$5x - 25 = 14 - 7y = 35z \Rightarrow \frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z-0}{1/35}$ $\Rightarrow \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z-0}{1} \dots\dots\dots(1)$ $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = 7\hat{i} - 5\hat{j} + \hat{k}$ <p>Required Vector equation:</p> $\vec{r} = \vec{a} + \lambda\vec{b} \Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$ <p>Required Cartesian equation:</p> $\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$ <p style="text-align: center;"><b>OR</b></p> $\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$ $\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$ $\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$ $ \vec{b}_1 \times \vec{b}_2  = \sqrt{9 + 1 + 49} = \sqrt{59}$ $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 10$	<p>0.5</p> <p>0.5</p> <p>1</p> <p>1</p> <p>0.5</p> <p>1</p> <p>0.5</p> <p>0.5</p>



	$= \int_0^{\pi} \frac{(\pi-x) \sin x}{1+\cos^2 x} dx \dots \dots \dots (2)$	0.5
	(1) + (2) $\Rightarrow$	
	$2I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x}$	1
	Put $t = \cos x \Rightarrow dt = -\sin x dx$	0.5
	$x = 0 \Rightarrow t = 1; x = \pi \Rightarrow t = -1$	
	$2I = \int_{-1}^1 \frac{\pi dx}{1+t^2} \Rightarrow 2I = \pi(\tan^{-1}t)_{-1}^1$	1
	$= \pi(\tan^{-1}(1) - \tan^{-1}(-1))$	0.5
	$2I = \pi \left( \frac{\pi}{4} + \frac{\pi}{4} \right) = \frac{\pi^2}{2} \Rightarrow I = \frac{\pi^2}{4}$	1
	<b>OR</b>	
	B) $I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$	0.5
	Put $t = \sin x - \cos x \Rightarrow dt = (\cos x + \sin x) dx$	1
	And $\sin 2x = 1 - (\sin x - \cos x)^2$	
	$x = \frac{\pi}{6} \Rightarrow t = \frac{1-\sqrt{3}}{2}; x = \frac{\pi}{3} \Rightarrow t = \frac{\sqrt{3}-1}{2}$	0.5
	$I = \int_{-(\sqrt{3}-1)/2}^{(\sqrt{3}-1)/2} \frac{dt}{\sqrt{1-t^2}}$	1
	$\Rightarrow I = 2 \int_0^{(\sqrt{3}-1)/2} \frac{dt}{\sqrt{1-t^2}} = 2(\sin^{-1}t)_0^{(\sqrt{3}-1)/2}$	1
	$\Rightarrow I = 2 \left( \sin^{-1} \left( \frac{\sqrt{3}-1}{2} \right) - 0 \right) = 2 \sin^{-1} \left( \frac{\sqrt{3}-1}{2} \right)$	1
33	$l_1: \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2} \Rightarrow l_1: \frac{x-1}{-3} = \frac{y-2}{p/7} = \frac{z-3}{2}$	0.5
	$l_2: \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5} \Rightarrow l_2: \frac{x-1}{3p/-7} = \frac{y-5}{1} = \frac{z-6}{-5}$	0.5
	d.r's of $l_1 = (-3, p/7, 2)$ , d.r's of $l_2 = \left(-\frac{3p}{7}, 1, -5\right)$	0.5
	$l_1 \perp l_2 \Rightarrow (-3) \left(-\frac{3p}{7}\right) + \left(\frac{p}{7}\right) (1) + (2)(-5) = 0$	
	$\Rightarrow p = 7$	1
	$\therefore$ d.r's of $l_1 = (-3, 1, 2)$	
	$\vec{a} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ , $\vec{b} = -3\hat{i} + \hat{j} + 2\hat{k}$	0.5

	<p>Required vector in Vector form: <math>\vec{r} = \vec{a} + \lambda\vec{b}</math></p> <p><math>\Rightarrow \vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 2\hat{k})</math></p> <p>Required vector in Cartesian form: <math>\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}</math></p>	1 1
34	<p><math>A = \begin{bmatrix} 1 &amp; -1 &amp; 0 \\ 2 &amp; 3 &amp; 4 \\ 0 &amp; 1 &amp; 2 \end{bmatrix}, B = \begin{bmatrix} 2 &amp; 2 &amp; -4 \\ -4 &amp; 2 &amp; -4 \\ 2 &amp; -1 &amp; 5 \end{bmatrix}</math></p> <p><math>BA = \begin{bmatrix} 2 &amp; 2 &amp; -4 \\ -4 &amp; 2 &amp; -4 \\ 2 &amp; -1 &amp; 5 \end{bmatrix} \times \begin{bmatrix} 1 &amp; -1 &amp; 0 \\ 2 &amp; 3 &amp; 4 \\ 0 &amp; 1 &amp; 2 \end{bmatrix} = \begin{bmatrix} 6 &amp; 0 &amp; 0 \\ 0 &amp; 6 &amp; 0 \\ 0 &amp; 0 &amp; 6 \end{bmatrix} = 6I</math></p> <p><math>\Rightarrow A^{-1} = \frac{B}{6}</math></p> <p>Given that</p> <p><math>x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7.</math></p> <p><math>A = \begin{bmatrix} 1 &amp; -1 &amp; 0 \\ 2 &amp; 3 &amp; 4 \\ 0 &amp; 1 &amp; 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}</math></p> <p><math>A \cdot X = D \Rightarrow X = A^{-1} \cdot D</math></p> <p><math>X = \frac{1}{6} \begin{bmatrix} 2 &amp; 2 &amp; -4 \\ -4 &amp; 2 &amp; -4 \\ 2 &amp; -1 &amp; 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}</math></p> <p><math>x = 2; y = -1; z = 4</math></p>	1 0.5 1 1 1.5
35	<p>A) <math>(x dy - y dx)y \sin\left(\frac{y}{x}\right) = (y dx + x dy)x \cos\left(\frac{y}{x}\right)</math></p> <p><math>\frac{dy}{dx} = \frac{y^2 \sin(y/x) + xy \cos(y/x)}{xy \sin(y/x) - x^2 \cos(y/x)}</math></p> <p>This is a homogeneous differential equation.</p> <p>Put <math>y = vx</math></p> <p><math>\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}</math></p> <p><math>v + x \frac{dv}{dx} = \frac{v^2 \sin v + v \cos v}{v \sin v - \cos v} \Rightarrow x \frac{dv}{dx} = \frac{2v \cos v}{v \sin v - \cos v}</math></p> <p><math>\Rightarrow \int \frac{v \sin v - \cos v}{2v \cos v} dv = \int \frac{dx}{x}</math></p> <p><math>\Rightarrow \frac{1}{2} \int \tan v dv - \frac{1}{2} \int \frac{1}{v} dv = \int \frac{dx}{x}</math></p> <p><math>\Rightarrow \frac{1}{2} (\log \sec v  - \log v ) = \log x  + \log C_1</math></p> <p><math>\Rightarrow \log \left  \frac{\sec v}{v} \right  = \log 2x^2 C_1  \Rightarrow \frac{\sec v}{v} = 2x^2 C_1</math></p> <p><math>\Rightarrow \frac{\sec(y/x)}{y/x} = Cx^2 \Rightarrow \sec(y/x) = Cxy</math></p>	1 0.5 1 0.5 0.5 0.5 0.5

	<b>OR</b>	
	$\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0 \Rightarrow \frac{dy}{dx} = -\frac{(y^2 + y + 1)}{x^2 + x + 1}$	0.5
	$\Rightarrow \frac{dy}{y^2 + y + 1} = -\frac{dx}{x^2 + x + 1}$	0.5
	$\Rightarrow \frac{dy}{\left(y + \frac{1}{2}\right)^2 + \frac{3}{4}} = -\frac{dx}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$	0.5
	$\Rightarrow \int \frac{dy}{\left(y + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = -\int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$	
	$\Rightarrow \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{y + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) = C$	1
	$\Rightarrow \tan^{-1} \left( \frac{2y+1}{\sqrt{3}} \right) + \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) = \frac{\sqrt{3}}{2} C$	
	$\Rightarrow \tan^{-1} \left( \frac{\frac{2y+1}{\sqrt{3}} + \frac{2x+1}{\sqrt{3}}}{1 - \frac{2y+1}{\sqrt{3}} \cdot \frac{2x+1}{\sqrt{3}}} \right) = \frac{\sqrt{3}}{2} C \Rightarrow \frac{\frac{2y+1}{\sqrt{3}} + \frac{2x+1}{\sqrt{3}}}{1 - \frac{2y+1}{\sqrt{3}} \cdot \frac{2x+1}{\sqrt{3}}} = \tan \left( \frac{\sqrt{3}}{2} C \right)$	1
	$\Rightarrow \frac{3/\sqrt{3}(2y+2x+2)}{3-4xy-2y-2x-1} = \tan \left( \frac{\sqrt{3}}{2} C \right)$	0.5
	$\frac{(y+x+1)}{(1-x-y-2xy)} = \frac{\sqrt{3}}{3} \tan \left( \frac{\sqrt{3}}{2} C \right)$	0.5
	$\Rightarrow \frac{(y+x+1)}{(1-x-y-2xy)} = A, \text{ where } A = \frac{\sqrt{3}}{3} \tan \left( \frac{\sqrt{3}}{2} C \right)$	0.5
	$\Rightarrow y+x+1 = A(1-x-y-2xy)$	0.5
	<b>SECTION E</b>	
36	(i) $\frac{dy}{dx} = \frac{d}{dx} \left( 10x - \frac{1}{2}x^2 \right) = 10 - x$	1
	(ii) At the point of maximum height $\frac{dy}{dx} = 0 \Rightarrow 10 - x = 0 \Rightarrow x = 10$ days	1
	(iii) $\frac{d^2y}{dx^2} = -1 < 0$ for $x = 10$	1
	$\therefore$ Maximum height $y$ (at $x = 10$ ) = $100 - 50 = 50$ cm	1
	<b>OR</b>	
	Height after 2 days $y$ (at $x = 2$ ) = $10 \times 2 - \frac{1}{2} \times 4 = 18$ cm	1+1

37	<p>(i) <math>f_1(x) = x^2 + 1</math> is not one-one, because  <math>f(1) = f(-1) = 2</math>, even though <math>1 \neq -1</math>.  Hence, different elements of domain give the same image.</p> <p>(ii) For <math>f_2(x) = 4x^2 - 3</math>, the range is <math>[-3, \infty)</math>.  Hence, if the function is onto,  Codomain <math>Y = [-3, \infty)</math>.</p> <p>(iii) For <math>f_3(x) = 2x - 5</math>:  One-one: If <math>f(x_1) = f(x_2)</math>, then <math>2x_1 - 5 = 2x_2 - 5 \Rightarrow x_1 = x_2</math>.  Hence, it is one-one.  Onto: For any <math>y \in \mathbb{R}</math>, there exists <math>x = \frac{y+5}{2} \in \mathbb{R}</math>. Hence, it is onto.  Therefore, <math>f_3(x) = 2x - 5</math> is bijective.</p> <p style="text-align: center;"><b>OR</b></p> <p>For <math>f_1(x) = x^2 + 1</math>:</p> <ul style="list-style-type: none"> <li>• Range: <math>[1, \infty)</math></li> <li>• Codomain: <math>\mathbb{R}</math></li> </ul> <p>Since not every real number is an image (e.g., 0 not in range), <math>f_1(x)</math> is not onto when codomain is <math>\mathbb{R}</math>.</p>	0.5 0.5  0.5 0.5  1  1   1  1
38	<p>(i) Required Probability =  <math>P(D/A)P(A) + P(D/B)P(B) + P(D/C)P(C) = 0.047</math></p> <p>(ii) The probability = 1 – probability that the form has a mistake and is processed by Ram  <math>= 1 - (30/47) = 17/47</math></p>	1+1  1+1

\*\*\*\*\*