



COMMON PRE-BOARD EXAMINATION

MATHEMATICS - Code No. 041

Class: XII (2025-26) -SET - 1



Time Allowed: 3 hours

ANSWER KEY

Maximum Marks: 80

NO	SECTION A	MARKS
----	-----------	-------

1	(d) $\left[0, \frac{2}{3}\right]$	1	11	(c) $-\cos x + C$	1
2	(a) Skew symmetric matrix	1	12	(c) $\sqrt{2}$	1
3	(b) 3	1	13	(a) $\frac{-2}{3}$	1
4	(c) 1/8	1	14	(c) -4	1
5	(d) 40	1	15	(a) $\sqrt{507}$	1
6	(d) $ A \in [2, 4]$	1	16	(b) The open half plane not containing origin.	1
7	(b) 2	1	17	(b) (0, 8)	1
8	(c) $\sec^2 y \tan y$	1	18	(c) not defined	1
9	(c) 12 cm ² /sec	1	19	(c) A is true and R is false.	1
10	(b) $\sec x$	1	20	(d) A is false but R is true.	1

SECTION B		
21	<p>A) $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$ $\Rightarrow 3 \times \pi/4 + 2 \times \pi/6 + \pi/2$ $\Rightarrow \frac{19\pi}{12}$</p> <p style="text-align: center;">OR</p> <p>B) $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$ $\Rightarrow \frac{\cos x}{1-\sin x} = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} - 2\sin \frac{x}{2} \cos \frac{x}{2}} = \frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} = \frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}}$ $= \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$</p>	<p>1.5</p> <p>0.5</p> <p>1</p> <p>0.5</p>

	$\tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) = \frac{\pi}{4} + \frac{x}{2}$	0.5
22	$f(0) = 3$ $\text{LHL} = \lim_{x \rightarrow 0^-} \frac{kx}{-x} = -k$ $\text{RHL} = \lim_{x \rightarrow 0^+} 3 = 3$ Since f is continuous at $x = 0$, $\text{LHL} = \text{RHL} = f(0) \Rightarrow -k = 3 \Rightarrow k = -3$	0.5 0.5 0.5 0.5
23	$y = 5e^{7x} + 6e^{-7x}$ $\frac{dy}{dx} = 35e^{7x} - 42e^{-7x}$ $\frac{d^2y}{dx^2} = 35 \cdot 7e^{7x} - 42 \cdot (-7)e^{-7x} = 49(5e^{7x} + 6e^{-7x}) = 49y$ $\frac{d^2y}{dx^2} = 49y$	0.5 1 0.5
24	$A) \int \frac{2^{x+1} - 5^{x-1}}{10^x} dx$ $\frac{2^{x+1}}{10^x} = 2 \cdot 5^{-x}, \frac{5^{x-1}}{10^x} = \frac{1}{5} \cdot 2^{-x}$ $\int \left(2 \cdot 5^{-x} - \frac{1}{5} \cdot 2^{-x}\right) dx = (-2) \cdot \frac{5^{-x}}{\log 5} - (-1) \cdot \frac{1}{5} \cdot \frac{2^{-x}}{\log 2} + C$ $= -\frac{2}{5^x \cdot \log 5} + \frac{1}{5 \cdot 2^x \cdot \log 2} + C$ <p style="text-align: center;">OR</p> $B) y = \sin x, [0, 2\pi]$ Required Area = $\int_0^\pi \sin x dx + \left \int_\pi^{2\pi} \sin x dx \right $ $= (-\cos x)_0^\pi + (-\cos x)_\pi^{2\pi} $ $= -(\cos \pi - \cos 0) + -(\cos 2\pi - \cos \pi) $ $= -(-1 - 1) + -(1 + 1) = 2 + 2 = 4 \text{ sq. unit}$	0.5 1 0.5 0.5 0.5 0.5 0.5
25	$\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}, \vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ $\vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$ $(\vec{b} + \vec{c}) \cdot \vec{a} = 6 - 2 + 2 = 6, \vec{a} = \sqrt{4 + 4 + 2} = 3$ Projection of $(\vec{b} + \vec{c})$ on $\vec{a} = \frac{(\vec{b} + \vec{c}) \cdot \vec{a}}{ \vec{a} } = \frac{6}{3} = 2$	0.5 0.5 + 0.5 0.5

	SECTION C	
26	$A) x = \cos t(3 - 2\cos^2 t) \Rightarrow x = \cos 3t \Rightarrow \frac{dx}{dt} = -3\sin 3t$	1
	$y = \sin t(3 - 2\sin^2 t) \Rightarrow y = \sin 3t \Rightarrow \frac{dy}{dt} = 3\cos 3t$	1
	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\cos 3t}{-3\sin 3t} = -\cot 3t$	0.5
	$t = \frac{\pi}{4} \Rightarrow \frac{dy}{dx} = -\cot \frac{3\pi}{4} = -(-1) = 1$	0.5
	OR	
	$B) y = (\sin x)^x + \sin^{-1}\sqrt{x}; u = (\sin x)^x; v = \sin^{-1}\sqrt{x}$	
$u = (\sin x)^x \Rightarrow \log u = x \log(\sin x)$	0.5	
$\frac{du}{dx} = (\sin x)^x (\log(\sin x) + x \cot x)$	1	
$v = \sin^{-1}\sqrt{x} \Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x-x^2}}$	1	
$\frac{dy}{dx} = (\sin x)^x (\log(\sin x) + x \cot x) + \frac{1}{2\sqrt{x-x^2}}$	0.5	
27	$f(x) = \sin^2 x - \cos x, x \in [0, \pi]$	
	$f'(x) = 2\sin x \cdot \cos x + \sin x$	1
	$f'(x) = 0 \Rightarrow 2\sin x \cdot \cos x + \sin x = 0 \Rightarrow \sin x(2\cos x + 1) = 0$	
	$\sin x = 0 \Rightarrow x = 0, \pi$	1
	$\cos x = -1/2 \Rightarrow x = 2\pi/3$	
	$f(0) = -1, f(2\pi/3) = 5/4, f(\pi) = 1$	
Absolute maximum = 5/4 at $x = 2\pi/3$	0.5	
Absolute minimum = -1 at $x = 0$	0.5	
28	$A) I = \int \frac{dx}{\sqrt{\sin^3 x \cos(x-a)}} = \int \frac{dx}{\sqrt{\sin^3 x (\cos x \cdot \cos a + \sin x \cdot \sin a)}}$	0.5
	$\int \frac{dx}{\sin^2 x \sqrt{\cot x \cdot \cos a + \sin a}} = \int \frac{\operatorname{cosec}^2 x \cdot dx}{\sqrt{\cot x \cdot \cos a + \sin a}}$	1
	Put, $t = \cot x \cdot \cos a + \sin a \Rightarrow \frac{dt}{dx} = -\operatorname{cosec}^2 x \cdot \cos a$	0.5
	$\frac{-dt}{\cos a} = \operatorname{cosec}^2 x dx$	

	$I = \frac{-1}{\cos a} \int \frac{dt}{\sqrt{t}} = -\frac{1}{\cos a} \times 2\sqrt{t} \Rightarrow I = -\frac{2\sqrt{\cot x \cdot \cos a + \sin a}}{\cos a} + C$	1
	<p>B) OR</p> <p> $x^2 + y^2 = 4, x = \sqrt{3}y$ $x^2 + y^2 = 4 \Rightarrow y = \sqrt{4 - x^2}$ $\Rightarrow x = \pm\sqrt{3}$ </p> <p>Required area of the shaded region =</p> $\int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{\sqrt{3}}^2 \sqrt{4 - x^2} dx$ $= \frac{\sqrt{3}}{2} + \left(\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right)_{\sqrt{3}}^2$ $= \frac{\sqrt{3}}{2} + (0 + 2\sin^{-1}1) - \left(\frac{\sqrt{3}}{2} \times 1 + 2\sin^{-1} \frac{\sqrt{3}}{2} \right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$	0.5
		1
		1
		1
		0.5
29	<p>A) $5x - 25 = 14 - 7y = 35z \Rightarrow \frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z-0}{1/35}$</p> <p>$\Rightarrow \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z-0}{1} \dots\dots\dots(1)$</p> <p>$\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = 7\hat{i} - 5\hat{j} + \hat{k}$</p> <p>Required Vector equation:</p> <p>$\vec{r} = \vec{a} + \lambda\vec{b} \Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$</p> <p>Required Cartesian equation:</p> <p>$\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$</p>	0.5
		0.5
		1
		1
	<p>B) OR</p> <p>$\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$</p> <p>$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$</p> <p>$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$</p> <p>$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$</p> <p>$\vec{b}_1 \times \vec{b}_2 = \sqrt{9 + 1 + 49} = \sqrt{59}$</p> <p>$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 10$</p> <p>$d = \left \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{ \vec{b}_1 \times \vec{b}_2 } \right = \frac{10}{\sqrt{59}}$</p>	0.5
		1
		0.5
		0.5
		0.5

<p>30</p>	<p>Minimize: $z = 5x + 10y$</p> <p>$x + 2y \leq 120 \dots(1)$</p> <p>$x + y \geq 60 \dots(2)$</p> <p>$x - 2y \geq 0 \dots(3)$</p> <p>$x, y \geq 0$</p> <p>$A = (60, 0), B = (120, 0)$</p> <p>Solving (1) & (3) $\Rightarrow C = (60, 30)$</p> <p>Solving (2) & (3) $\Rightarrow D = (40, 20)$</p> <p>$A = (60, 0) \Rightarrow Z = 300$</p> <p>$B = (120, 0) \Rightarrow Z = 600$</p> <p>$C = (60, 30) \Rightarrow Z = 600$</p> <p>$D = (40, 20) \Rightarrow Z = 400$</p> <p>Minimum Value = 300, when $x = 60$ & $y = 0$</p>	<p>Graph- 1.5</p> <p>1</p> <p>0.5</p>
<p>31</p>	<p>$S = \{bb, bg, gb, gg\}$</p> <p>a) A : A be that both children are girls</p> <p>$\Rightarrow A = \{gg\} \Rightarrow P(A) = 1/4$</p> <p>B: youngest child is a girl</p> <p>$\Rightarrow B = \{gg, bg\} \Rightarrow P(B) = \frac{2}{4} = \frac{1}{2}$</p> <p>$A \cap B = \{gg\} \Rightarrow P(A \cap B) = \frac{1}{4}$</p> <p>$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/2} = \frac{1}{2}$</p> <p>b) C : at least one child is a girl</p> <p>$C = \{bg, gb, gg\} \Rightarrow P(C) = 3/4$</p> <p>$A \cap C = \{gg\} \Rightarrow P(A \cap C) = 1/4$</p> <p>$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{1/4}{3/4} = 1/3$</p>	<p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p>
<p>SECTION D</p>		
<p>32</p>	<p>$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$</p> <p>$BA = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$</p>	<p>1</p>

	$\Rightarrow A^{-1} = \frac{B}{6}$ <p>Given that</p> $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7.$ $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $A \cdot X = D \Rightarrow X = A^{-1} \cdot D$ $X = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $x = 2; y = -1; z = 4$	<p>0.5</p> <p>1</p> <p>1</p> <p>1.5</p>
33	<p>A) $I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx \dots \dots \dots (1)$</p> $I = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx$ $= \int_0^{\pi} \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx \dots \dots \dots (2)$ <p>(1) + (2) \Rightarrow</p> $2I = \int_0^{\pi} \frac{\pi \sin x}{1 + \cos^2 x}$ <p>Put $t = \cos x \Rightarrow dt = -\sin x dx$</p> <p>$x = 0 \Rightarrow t = 1; x = \pi \Rightarrow t = -1$</p> $2I = \int_{-1}^1 \frac{\pi dx}{1 + t^2} \Rightarrow 2I = \pi (\tan^{-1} t)_{-1}^1$ $= \pi (\tan^{-1}(1) - \tan^{-1}(-1))$ $2I = \pi \left(\frac{\pi}{4} + \frac{\pi}{4} \right) = \frac{\pi^2}{2} \Rightarrow I = \frac{\pi^2}{4}$ <p style="text-align: center;">OR</p> <p>B) $I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$</p> <p>Put $t = \sin x - \cos x \Rightarrow dt = (\cos x + \sin x) dx$</p> <p>And $\sin 2x = 1 - (\sin x - \cos x)^2$</p> <p>$x = \frac{\pi}{6} \Rightarrow t = \frac{1 - \sqrt{3}}{2}; x = \frac{\pi}{3} \Rightarrow t = \frac{\sqrt{3} - 1}{2}$</p> $I = \int_{-(\sqrt{3}-1)/2}^{(\sqrt{3}-1)/2} \frac{dt}{\sqrt{1 - t^2}}$	<p>0.5</p> <p>0.5</p> <p>1</p> <p>0.5</p> <p>1</p> <p>1</p> <p>0.5</p> <p>1</p>

	$\Rightarrow \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) = \frac{\sqrt{3}}{2}C$ $\Rightarrow \tan^{-1}\left(\frac{\frac{2y+1}{\sqrt{3}} + \frac{2x+1}{\sqrt{3}}}{1 - \frac{2y+1}{\sqrt{3}} \cdot \frac{2x+1}{\sqrt{3}}}\right) = \frac{\sqrt{3}}{2}C \Rightarrow \frac{\frac{2y+1}{\sqrt{3}} + \frac{2x+1}{\sqrt{3}}}{1 - \frac{2y+1}{\sqrt{3}} \cdot \frac{2x+1}{\sqrt{3}}} = \tan\left(\frac{\sqrt{3}}{2}C\right)$ $\Rightarrow \frac{3/\sqrt{3}(2y+2x+2)}{3-4xy-2y-2x-1} = \tan\left(\frac{\sqrt{3}}{2}C\right)$ $\frac{(y+x+1)}{(1-x-y-2xy)} = \frac{\sqrt{3}}{3} \tan\left(\frac{\sqrt{3}}{2}C\right)$ $\Rightarrow \frac{(y+x+1)}{(1-x-y-2xy)} = A, \text{ where } A = \frac{\sqrt{3}}{3} \tan\left(\frac{\sqrt{3}}{2}C\right)$ $\Rightarrow y+x+1 = A(1-x-y-2xy)$	1 0.5 0.5 0.5
35	$l_1: \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2} \Rightarrow l_1: \frac{x-1}{-3} = \frac{y-2}{p/7} = \frac{z-3}{2}$ $l_2: \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5} \Rightarrow l_2: \frac{x-1}{3p/-7} = \frac{y-5}{1} = \frac{z-6}{-5}$ <p>d.r's of $l_1 = (-3, p/7, 2)$, d.r's of $l_2 = \left(-\frac{3p}{7}, 1, -5\right)$</p> $l_1 \perp l_2 \Rightarrow (-3)\left(-\frac{3p}{7}\right) + \left(\frac{p}{7}\right)(1) + (2)(-5) = 0$ $\Rightarrow p = 7$ <p>\therefore d.r's of $l_1 = (-3, 1, 2)$</p> $\vec{a} = 3\hat{i} + 2\hat{j} - 4\hat{k}, \vec{b} = -3\hat{i} + \hat{j} + 2\hat{k}$ <p>Required vector in Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$</p> $\Rightarrow \vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 2\hat{k})$ <p>Required vector in Cartesian form: $\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$</p>	0.5 0.5 0.5 1 0.5 1 1
SECTION E		
36	<p>(a) $f_1(x) = x^2 + 1$ is not one-one, because $f(1) = f(-1) = 2$, even though $1 \neq -1$. Hence, different elements of domain give the same image.</p> <p>(b) For $f_2(x) = 4x^2 - 3$, the range is $[-3, \infty)$. Hence, if the function is onto, Codomain $Y = [-3, \infty)$.</p> <p>(c) For $f_3(x) = 2x - 5$:</p>	0.5 0.5 0.5 0.5

	<p>One-one: If $f(x_1) = f(x_2)$, then $2x_1 - 5 = 2x_2 - 5 \Rightarrow x_1 = x_2$. Hence, it is one-one.</p> <p>Onto: For any $y \in \mathbb{R}$, there exists $x = \frac{y+5}{2} \in \mathbb{R}$. Hence, it is onto.</p> <p>Therefore, $f_3(x) = 2x - 5$ is bijective.</p> <p style="text-align: center;">OR</p> <p>(c) For $f_1(x) = x^2 + 1$:</p> <ul style="list-style-type: none"> • Range: $[1, \infty)$ • Codomain: \mathbb{R} <p>Since not every real number is an image (e.g., 0 not in range), $f_1(x)$ is not onto when codomain is \mathbb{R}.</p>	<p style="text-align: right;">1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1</p>
37	<p>(a) $\frac{dy}{dx} = \frac{d}{dx} \left(10x - \frac{1}{2}x^2 \right) = 10 - x$</p> <p>(b) At the point of maximum height $\frac{dy}{dx} = 0 \Rightarrow 10 - x = 0 \Rightarrow x = 10$ days</p> <p>(c) $\frac{d^2y}{dx^2} = -1 < 0$ for $x = 10$</p> <p>\therefore Maximum height y (at $x = 10$) = $100 - 50 = 50$ cm</p> <p style="text-align: center;">OR</p> <p>(c) Height after 2 days y (at $x = 2$) = $10 \times 2 - \frac{1}{2} \times 4 = 18$ cm</p>	<p style="text-align: right;">1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1+1</p>
38	<p>(a) Required Probability = $P(D/A)P(A) + P(D/B)P(B) + P(D/C)P(C) = 0.047$</p> <p>(b) The probability = 1 – probability that the form has a mistake and is processed by Ram $= 1 - (30/47) = 17/47$</p>	<p style="text-align: right;">1+1</p> <p style="text-align: right;">1+1</p>
