COMMON PRE-BOARD EXAMINATION 2022-23

CLASS: X

SUBJECT: MATHEMATICS (041)

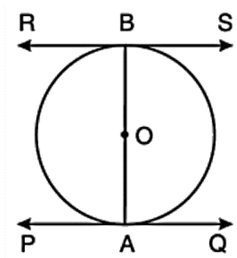
ANSWER KEY

Q. No	**SECTION A**	Marks
1	(a)2	1
2	(c) 0 and 8	1
3	(b) – 10	1
4	(a)14	1
5	(d) (3,0)	1
6	(c) 38	1
7	(c) $\frac{a}{\sqrt{b^2 - a^2}}$	1
8	(b)1	1
9	(a)1	1
10	(b)Similar but not congruent	1
11	(b)65^0	1
12	(a)77 m^2	1
13	(d)1:2	1
14	(b)52	1
15	(a)14 cm^2	1
16	(d)$\frac{4}{3}\pi a^3$	1
17	(d)$\frac{2}{3}$	1
18	(a)$2\frac{1}{4}$	1
19	(a)Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).	1
20	(d)Assertion (A) is false but Reason (R) is true.	1
	SECTION B (2 marks each)	
21	x + y = 2125 --(1mark) 3x + 4y = 6400 --(1mark)	**(1)** **(1)**
22	In ΔPQR, PM/QM = 4/4.5	

	PN/NR = 4/4.5 PM/QM = PN/NR Hence MN || to QR, $\angle PMN = \angle PQR$(Corresponding angle)	 **(½)** **(½)** **(1)**
23	$\angle OPQ = 90° - 70° = 20°$ OP=OQ (radius of circle) $\angle OQP = \angle OPQ = 20°$ $\angle POQ = 180° - (\angle OPQ + \angle OQP)$ $= 180° - (20° + 20°)$ $= 180° - 40° = 140°$	**(½)** **(½)** **(½)** **(½)**
24	radius (r) = Circumference/2π = 22/ (2 × 22/7) = (22 × 7)/ (2 × 22) = 7/2 cm Therefore, the area of a quadrant = 1/4 × πr^2 = 1/4 × 22/7 × 7/2 × 7/2 = $\frac{77}{8}$ cm^2	**(½)** **(½)** **(½)** **(½)**
OR	In 5 minutes, minute hand will rotate $\frac{360°}{60} \times 5 = 30°$ Area of sector of angle θ = $\frac{\theta}{360} . \pi . r^2$ Area of sector of 30° = $\frac{30}{360} \times \frac{22}{7} \times 14 \times 14$ = 51.33 cm^2	**(½)** **(½)** **(½)** **(½)**
25	Consider LHS=2 $(cos^2 45° + tan^2 60°) - 6 (sin^2 45° - tan^2 30°)$ $=2[(\frac{1}{\sqrt{2}})^2 + (\sqrt{3})^2] - 6[(\frac{1}{\sqrt{2}})^2 - (\frac{1}{\sqrt{3}})^2]$ $= 2\left[\left(\frac{1}{2}\right) + 3\right] - 6[\left(\frac{1}{2}\right) - \left(\frac{1}{3}\right)]$ $=2\left(\frac{7}{2}\right) - 6\left(\frac{1}{6}\right)$ $= 7 - 1 = 6 = RHS$	 **(½)** **(½)** **(½)** **(½)**

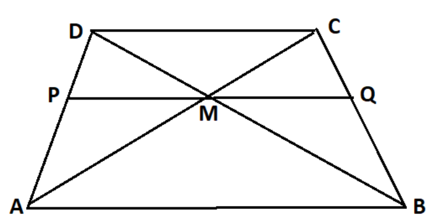
0R	LHS = $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$ $=\frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$ $=\frac{\sin^2\theta+\cos^2\theta+2sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$ $=\frac{1+1}{\sin^2\theta-\cos^2\theta}$ $=\frac{2}{\sin^2\theta-\cos^2\theta}$ $=\frac{2}{(1-cos^2\theta)-cos\theta}=\frac{2}{1-2cos^2\theta}$	**(½)** **(½)** **(½)** **(½)**
	SECTION C (3 marks each)	
26	Let $(3-\sqrt{5})$ be a rational number. $(3-\sqrt{5})=\mathrm{x}$, where $\mathrm{x}=\frac{\mathrm{p}}{\mathrm{q}}$, p and q are coprime and $\mathrm{q}\neq 0$. $\therefore \sqrt{5}=3-\mathrm{x}$ $\because$ x is an integer, $3-\mathrm{x}$ is an integer Hence $\sqrt{5}$ is rational which is not possible. Hence our assumption is wrong. $(3-\sqrt{5})$ is irrational.	**(½)** **(½)** **(½)** **(½)** **(½)** **(½)**
27	When roots are given, quadratic polynomial is of the form $x^2-(\alpha+\beta)x+\alpha\beta$ $Also\ \alpha+\beta=\frac{5}{2}\ and$ $\alpha.\beta=\frac{7}{2}$ $Here\ roots\ are\ 2\alpha\ and\ 2\beta$ $\therefore$ the req: quadratic polynomial is $\mathrm{k}\{\mathrm{x}^2-(2\alpha+2\beta)\mathrm{x}+2\alpha.2\beta\}$ $=\ \mathrm{k}\{\mathrm{x}^2-2(\alpha+\beta)\mathrm{x}+4\alpha\beta\}$ $=k(x^2-5x+14)$	**(½)** **(½)** **(½)** **(½)** **(½)** **(½)**

<table>
<tr><td></td><td></td><td></td></tr>
<tr><td>28</td><td>2x + 3y = 2
<table><tr><td>x</td><td>1</td><td>-5</td></tr><tr><td>y</td><td>0</td><td>4</td></tr></table>Plotting the solutions on the cartesian plane
x – 2y = 8
<table><tr><td>x</td><td>8</td><td>0</td></tr><tr><td>y</td><td>0</td><td>- 4</td></tr></table>Plotting the solutions on the cartesian plane.
Solution for the pair of equations is x = 4 and y = – 2
Shading of the triangular region bounded by these lines and x – axis</td><td>(½)
(½)
(½)
(½)
(½)
(½)</td></tr>
<tr><td>OR</td><td>let the cost of one pencil be ₹ x and the cost of one pencil be ₹ y
$5x + 7y = 195 \dots\dots\dots (i)$
$7x + 5y = 153$ ………(ii)
Adding (i) and (ii), $12x + 12y = 348 \Rightarrow x + y = 29$ ………(iii)
Subtracting (i) from (ii),$2x - 2y = -42 \Rightarrow x - y = -21$ ………(iv)
Solving we get , the cost of each pencil is ₹ 4
and the cost of each pen is ₹ 25</td><td>(½)
(½)
(½)
(½)
(½)
(½)</td></tr>
<tr><td>29</td><td>LHS = $\frac{cosA}{1-tanA} + \frac{sinA}{1-cotA}$
$= \frac{cosA}{1-tanA} + \frac{sinAtanA}{tanA - 1}$
$= \frac{cosA}{1-tanA} - \frac{sinAtanA}{1- tanA}$
$= \frac{cosA - sinAtanA}{1-tanA}$
$= \frac{cos^2A - sin^2A/_{cosA}}{cosA - sinA/_{cosA}}$
$= \frac{(cosA - sinA)(cosA + sinA)}{cosA - sinA}$
$= cosA + sinA$</td><td>(½)
(½)
(½)
(½)
(½)
(½)</td></tr>
<tr><td>30</td><td>$A P = PB$ (Tangents from external point)
$AP = 4\ cm$</td><td>(½)</td></tr>
</table>

	$\angle PAB = 180° - 135° = 45°$ (Supplementary angles) $\angle ABP = \angle PAB = 45°$ (Opposite angles of equal sides) $\angle APB = 180° - 45° - 45° = 90°$. So Δ ABP is an isosceles right angled triangle Now in ΔABP we have $\sin 45 = \frac{AP}{AB}$ $ie\ \frac{1}{\sqrt{2}} = \frac{4}{AB}$ $\therefore AB = 4\sqrt{2}cm$	**(½)** **(½)** **(½)** **(½)** **(½)**
OR	**Given.** Let AB be a diameter of a given circle, and let PQ and RS be the tangent lines drawn to the circle at points A and B respectively.  **To prove.** $PQ \parallel RS$ **Proof.** $AB \perp PQ$ and $AB \perp RS$ $\Rightarrow \quad \angle PAB = 90°$ and $\quad \angle ABS = 90°$ $\Rightarrow \quad \angle PAB = \angle ABS$ $\Rightarrow \quad \boldsymbol{PQ \parallel RS}$ [$\because$ $\angle PAB$ and $\angle ABS$ are alternate angles]	**(½)** **(½)** **(½)** **(½)** **(½)** **(½)**
31	Since, Jacks, Queens and Kings of red colour are removed. Then, Total number of possible outcomes = 52−6=46 (i) Let E_1 be the event of getting a black king ∴ Favourable outcomes = King of spade and King of club.	

	No. of favourable outcomes = 2 $P(E_1) = \frac{2}{46} = \frac{1}{23}$ (ii) LetE_2 be the event of getting a card of red colour ∴ Favourable outcomes = 10 cards of heart and 10 cards of diamond. No. of favourable outcomes = 20 $P(E_2) = \frac{20}{46} = \frac{10}{23}$ (iii) Let E_3 be the event of getting a card of black colour ∴ Favourable outcomes = 13 cards of spade and 13 cards of club. No. of favourable outcomes = 26 $P(E_3) = \frac{26}{46} = \frac{13}{23}$	**(½)** **(½)** **(½)** **(½)** **(½)** **(½)**
32	Equation is $x^2 + px + 16 = 0$ where, a=1 , b=p, c=16 $b^2 - 4ac = 0$ $(p)^2 - 4(1)(16) = 0$ $p^2 - 64 = 0$ $p = \sqrt{64}$ $p = \pm 8$ Solution is $x = \frac{-b \pm \sqrt{D}}{2a}$	 **(½)** **(½)** **(½)** **(½)** **(½)+(½)**

	When p=8, $x^2+8x+16=0$ The solution is, $x=\frac{-(8)\pm\sqrt{0}}{2(1)}$, $x=\frac{-8}{2}$, $x=-4$ When p=-8, $x^2-8x+16=0$ The solution is, $x=\frac{-(-8)\pm\sqrt{0}}{2(1)}$, $x=\frac{8}{2}$, $x=4$ The roots of the equation is 4, -4.	**(½)** **(½)** **(½)** **(½)**
OR	Let the time taken by pipe of larger diameter be x hours and time taken by the pipe with smaller diameter be x + 10 hours. Now, $\frac{4}{x}+\frac{9}{x+10}=\frac{1}{2}$...........(i) $\Rightarrow x^2-16x-80=0$..........(ii) Solve eq (ii)……$(x-20)(x+4)=0$ $x=20, x=-4.$ The value of x can not be -ve so that the value of x=20. So that the larger diameter pipe fill the tank in 20 hours and smaller diameter pipe fill the tank in 30 hour	**(½)** **(1)** **(1)** **(½) + (½)** **(½)** **(½) + (½)**
33	Original volume of water in the cylindrical tub = Volume of Cylinder $=\pi r^2 h$ $=\frac{2}{7}\times 5^2\times 1.4$ $=22\times 25\times 1.4$ $=770\ cm^3$	**(½)** **(½)** **(½)**

	Given that Radius of hemisphere, R=2.1 cm and height of cone, h=4 cm Volume of solid immersed = Volume of cone+ Volume of hemisphere $=\frac{1}{3}\pi R^2 h+\frac{2}{3}\pi R^3$ $=\frac{1}{3}\pi R^2(h+2R)$ $=\frac{1}{3}\times\frac{22}{7}\times 2.1\times 2.1(4+2\times 2.1)$ $=\frac{1}{3}\times\frac{22}{7}\times 2.1\times 2.1\times 8.2$ $=37.884\ cm\ cube$ ∴ Volume of water displaced (removed)= $37.884\ cm\ cube$ Hence, the required volume of the water left in the cylindrical tub =770−37.884 =732.116 cm^3	**(½)** **(½)** **(½)** **(½)** **(½)** **(½)** **(½)**
34	Given, to prove, figure, construction Correct proof  In ΔABD, $\frac{DP}{PA}=\frac{DM}{MB}$ ---------(i) In ΔBDC, $\frac{DM}{MB}=\frac{CQ}{QB}$ -----------(ii) $from\ (i) and\ (ii)\ \frac{DP}{PA}=\frac{CQ}{QB}$	**(½)** **(½)** **(2)** **(½)** **(½)** **(½)** **(½)**

	Hence a line through the point of intersection of the diagonals and parallel to one of the parallel sides of the trapezium divides the non-parallel sides in the same ratio.	
OR	Given, To prove and figure In ΔQBC and Δ PAC, $\angle BCQ = \angle ACP$ $\angle QBC = \angle PAC$ $\therefore \Delta QBC \sim \Delta PAC$ $\frac{BC}{AC} = \frac{QB}{PA}$ $\frac{b}{a+b} = \frac{z}{x}$ $\therefore \frac{b}{z} = \frac{a+b}{x}$------------(i) Similarly, $\Delta ABQ \sim \Delta ACR$ $\frac{AB}{AC} = \frac{BQ}{CR}$ $\frac{a}{a+b} = \frac{z}{y}$ $\therefore \frac{a}{z} = \frac{a+b}{y}$------------(ii) $(i) + (ii) \Longrightarrow \frac{b}{z} + \frac{a}{z} = \frac{a+b}{x} + \frac{a+b}{y}$ $\frac{a+b}{z} = (a+b)(\frac{1}{x} + \frac{1}{y})$	**(½)** **(½)** **(½)** **(½)** **(½)** **(½)** **(½)** **(½)** **(½)** **(½)**

<table>
<tr><td></td><td>$\Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{1}{y}$</td><td></td></tr>
<tr><td>35</td><td>

Class interval	Frequency	Cumulative frequency
0 – 10	5	5
10 – 20	x	5+x
20 – 30	6	11+x
30 – 40	y	11+x+y
40 – 50	6	17+x+y
50 – 60	5	22+x+y
Total	40	22+x+y

We have x+y = 18 -------------(i)

Now Median = $l + \frac{\frac{n}{2} - cf}{f} \times h$

Here $l = 30$, h = 10 , $\frac{n}{2} = 20$, cf = 11+x , f = y

Now we have $31 = 30 + \frac{(20-11-x)}{y} \times 10$

$\Rightarrow 10x + y = 90$ ---------------(ii)

Solving (i) and (ii) we get $x = 8 \; and \; y = 10$

</td><td>

(½)

(½)

(½)x5=2 ½

(½)
(½)+(½)

</td></tr>
<tr><td></td><td>

SECTION E

3 case study questions of 4 marks each

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<tr><td>36</td><td>(i) $Distance\ OA = \sqrt{(0 - -2)^2 + (0 - 2)^2}$

$= 2\sqrt{2}$</td><td>**(½)**
(½)</td></tr>
<tr><td></td><td>(ii) $Distance\ AB = \sqrt{(-2 - -1)^2 + (-2 - 2)^2}$</td><td>**(½)**
(½)</td></tr>
</table>

	$= \sqrt{17}$	
	(iii) $D(x,y) = (\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2})$ $= (\frac{4\times -1 + 3\times -2}{4+3}, \frac{4\times -2 + 3\times 2}{4+3})$ $= (\frac{-10}{7}, \frac{-2}{7})$	**(½)** **(½)+(½)** **(½)**
OR	Distance AO $= 2\sqrt{2}$,Distance AB $= \sqrt{17}$ $Distance\ OB = \sqrt{(0 - -1)^2 + (0 - -2)^2}$ $= \sqrt{5}$ Since AB $\neq$ AO $\neq$ OB, its a scalene triangle.	**(½)** **(½)** **(½)** **(½)**
37	(i) a = 1500 and d = 200	**(½) + (½)**
	(ii) amount paid in the 15th installment, a_{15} = 1500 + 14x200 = ₹ 4300	**(½)** **(½)**
	(iii) 1500 + (n – 1)200 = 5500 (n – 1) 200 = 4000 n = 21	**(1)** **(½)** **(½)**
OR	2k + 1 – 11 = (3k – 1) – (2k + 1) 2k – 10 = k – 2 k = 8	**(1)** **(½)** **(½)**
38	(i) Let the distance of two cars from the tower be x and y $\tan 30 = \frac{3}{x} \Rightarrow x = 3\sqrt{3}$	**(½)**

	$tan60 = \frac{3}{y} \Rightarrow y = \sqrt{3}$ Distance between the cars = $4\sqrt{3}m$	**(½)**
	(ii) Angle of depression will increase as the car approaches the building.	**(1)**
	(iii) $Given\ \frac{opposite}{adjacent} = \frac{\sqrt{3}}{1}$ $\Rightarrow \tan\theta = \sqrt{3}$ $hence\ the\ angle\ of\ elevation\ of\ the\ sun = 60^0$	**(½)** **(1)** **(½)**
OR	If both height and base are increased by 10%, $$\tan\alpha = \frac{h + \frac{1}{10}h}{b + \frac{1}{10}b}$$ $$\tan\alpha = \frac{h}{b}$$ ie $\theta = \alpha$ So angle of elevation remains unchanged.	**(½)** **(½)** **(½)** **(½)**