INDIAN SCHOOL AL MAABELA

(ISO 9001: 2015 CERTIFIED INSTITUTION)

COMMON PRE BOARD EXAM EXAMINATION - 2022-2023

SUBJECT: MATHEMATICS (241)

ISAM/FR/SEC/MS/04

CLASS: X **Max. Marks: 80**

Date: **Time: 3 Hr**

MARKING SCHEME

1	The points given are (4, p) and (1, 0) By distance formula, 5 = √(4 − 1)2 + p 2 ⇒ 25 = (4 − 1)2 + p 2 ⇒ 25 = 32 + p2 ⇒ 25 = 9 + p2 ⇒ p 2 = 25 – 9 = 16 ⇒ p = ±4	1
2	a^3b^2	1
3	$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$	1
4	Total number of possible outcomes = 36 Now for the product of the numbers on the dice is prime number can be have in these possible ways = (1, 2), (2, 1), (1, 3), (3, 1), (5, 1), (1, 5) So, number of possible ways = 6 ∴ Required probability = 6/36 = 1/6	
5.	The given equations are: $x + 2y + 5 = 0$ $-3x - 6y + 1 = 0$ From the given equations we have: $\frac{a_1}{a_2} = \frac{1}{-3}; \frac{b_1}{b_2} = \frac{2}{-6} = \frac{1}{-3}; \frac{c_1}{c_2} = \frac{5}{1}$ $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Hence the given pair of equations have no solution.	
6	Here, a = 2, b = -7 and c = 3 p + q = -b/a = 7/2 and pq = c/a = 3/2 $p + q - pq = \frac{7}{2} - \frac{3}{2} = \frac{7-3}{2} = \frac{4}{2} = 2$	
7.	$\sin 2A = \frac{1}{2}\tan^2 45^0 = \frac{1}{2} \times 1^2 = \frac{1}{2} = \sin 30^0 \Rightarrow 2A = 30^0 \Rightarrow A = 15^0$	

8 $\cot C = \sqrt{3} = \cot 30°$, so, $\angle C = 30°$,

Hence, $\angle A = 60°$.

So, $\cos A \sin C + \sin A \cos C = \frac{1}{2}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}=\frac{1}{4}+\frac{3}{4}=\frac{1+3}{4}=\frac{4}{4}=1$

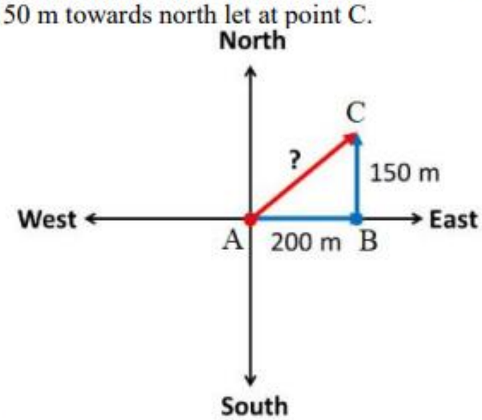
9 **Ans: (c)** $\frac{\sqrt{2}}{2}$

$\tan\theta+\cot\theta=2 \Rightarrow \tan\theta+\frac{1}{\tan\theta}=2 \;\Rightarrow \tan^2\theta-2\tan\theta+1=0$

$\Rightarrow (\tan\theta-1)^2=0 \Rightarrow \tan\theta=1=\tan 45^0 \;\Rightarrow \theta=45^0$

$Now, \sin^3\theta+\cos^3\theta=\sin^3 45^0+\cos^3 45^0=\left(\frac{1}{\sqrt{2}}\right)^3+\left(\frac{1}{\sqrt{2}}\right)^3=\frac{1}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

10 Let girl starting point is A and she goes to B towards east covering 200 m distance .
Now from B she moves 150 m towards north let at point C.



Then, using Pythagoras theorem we get,

$\Rightarrow AC = \sqrt{(AB^2 + BC^2)} \Rightarrow AC = \sqrt{(200^2 + 150^2)} \Rightarrow AC = \sqrt{(40000 + 22500)}$

$\Rightarrow AC = \sqrt{62500} \Rightarrow AC = 250$ m

11. **Ans: (c) 4.5 cm**

$DE \parallel AB \Rightarrow \frac{CD}{DA}=\frac{CE}{EB} \Rightarrow \frac{3}{DA}=\frac{4}{6} \Rightarrow DA=\frac{9}{2}=4.5cm$

12 **Ans: (b) 50⁰**

13. **Ans: (b) mode = 3 median – 2 mean**

14. **Ans: (c) 30 – 40**

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of Students	3	9	15	30	18	5

Highest frequency is 30 which belong to 30 – 40. Hence, Modal class is 30 – 40

15.	**Ans: (b)** $\frac{77}{8}$ **cm²** Let the radius of the circle be 'r' Circumference (C) = 22 cm ⇒ radius (r) = C/2π = 22/(2 × 22/7) = (22 × 7)/(2 × 22) = 7/2 cm Therefore, the area of a quadrant = 1/4 × πr^2 = 1/4 × 22/7 × 7/2 × 7/2 = 77/8 cm^2	
16.	**Ans: (c) k = 4** For a quadratic equation to have equal and real roots the discriminant should be equal to zero. D = 0. Now, D = $b^2 - 4ac$ ⇒ 0 = $(4)^2$ – 4 (1)(k) ⇒ 0 = 16 – 4k ⇒ 4k = 16 ⇒ k = 16/4 ⇒ k = 4	
17.	Ans: (d) 16 : 9 Let the radius of two spheres be r1 and r2. Given, the ratio of the volume of two spheres = 64 : 27 $\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$ Let the surface areas of the two spheres be S1 and S2. $\therefore \frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$	
18.	Ans: (b) 128 cm² Radius of circle =8 cm ⇒ Diameter = 8 × 2=16 cm The diameter of circle = diagonal of square =16 cm = a√2 ⇒ a = 16/√2 = 8√2 Area of square = $(\text{side})^2 = (8\sqrt{2})^2 = 128\ cm^2$	
19.	**Ans: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).** HCF x LCM = Product of two numbers $\Rightarrow LCM = \frac{90 \times 144}{18} = 5 \times 144 = 720$	
20.	Ans: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)	
	SECTION B	

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<td>21.</td>
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Ans: $\tan(A + B) = \sqrt{3} = \tan 60°$
$\Rightarrow A + B = 60°$ ……(i)
$\tan(A – B) = 1/\sqrt{3} = \tan 30°$
$\Rightarrow A – B = 30°$ …… (ii)
Adding equation (i) and (ii),
$2A = 90° \Rightarrow A = 45°$
Putting the value of A in equation (i),
$45° + B = 60°$
$\Rightarrow B = 60° - 45° \Rightarrow B = 15°$
OR
If $x\sin^3\theta + y\cos^3\theta = \sin\theta \cos\theta$ and $x\sin\theta = y\sin\theta$ then find $x^2 + y^2$.
Ans:
We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$
$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$
$\Rightarrow x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$
$\Rightarrow x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$
$\Rightarrow x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$
Now, $x\sin\theta = y\cos\theta \Rightarrow \cos\theta\sin\theta = y\cos\theta \Rightarrow y = \sin\theta$
Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$
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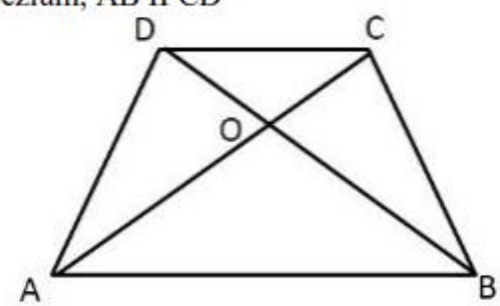
Ans: Given : ABCD is a trapezium, AB II CD


In ΔAOB and ΔCOD
$\angle OBA = \angle ODC$ ----eq.1 (alt.angles are equal)
$\angle OAB = \angle OCD$ ----eq.2 (alt.angles are equal)
Therefore, $\Delta AOB \sim \Delta COD$(A.A Similarity)
Hence, $\frac{OA}{OC} = \frac{OB}{OD}$
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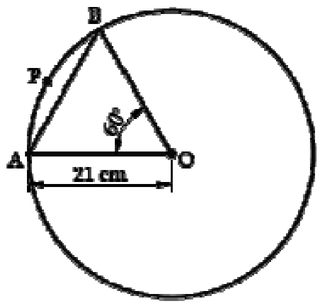
23.

Ans: We know that the minute hand completes one rotation in 1 hour or 60 minutes.
Length of the minute hand (r) = 14 cm
Area swept by minute hand in 1 minute = $\pi r^2/60$
Thus, area swept by minute hand in 5 minutes = $(\pi r^2/60) \times 5 = \pi r^2/12$
$= 1/12 \times 22/7 \times 14 \times 14 \text{ cm}^2$
$= 154/3 \text{ cm}^2$

OR

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find (i) the length of the arc (ii) area of the sector formed by the arc
Ans: Here, r = 21 cm, θ = 60°



Area of the segment APB = Area of sector AOPB - Area of ΔAOB
(i) Length of the Arc, APB = $\theta/360° \times 2\pi r$
$= 60°/360° \times 2 \times 22/7 \times 21$ cm
= 22 cm
(ii) Area of the sector, AOBP = $\theta/360° \times \pi r^2$
$= 60°/360° \times 22/7 \times 21 \times 21 \text{ cm}^2$
$= 231 \text{ cm}^2$

24.

Ans: $Let\ \angle APO = \theta$

$$\Rightarrow \sin\theta = \frac{OA}{OP} = \frac{1}{2} = \sin 30^0 \Rightarrow \theta = 30^0$$

$$\Rightarrow \angle APO = 2\theta = 2(30^0) = 60^0$$

$$Also, \angle PAB = \angle PBA = 60^0 \quad (\because PA = PB)$$

⇒ ΔAPB is an equilateral triangle.

25.

Ans: $3x + y - 1 = 0$
$(2k - 1)x + (k - 1)y - 2k - 1 = 0$

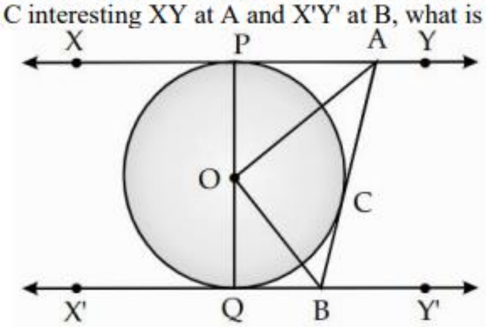
$$\frac{a_1}{a_2} = \frac{3}{2k-1}; \frac{b_1}{b_2} = \frac{1}{k-1}; \frac{c_1}{c_2} = \frac{-1}{-2k-1} = \frac{1}{2k+1}$$

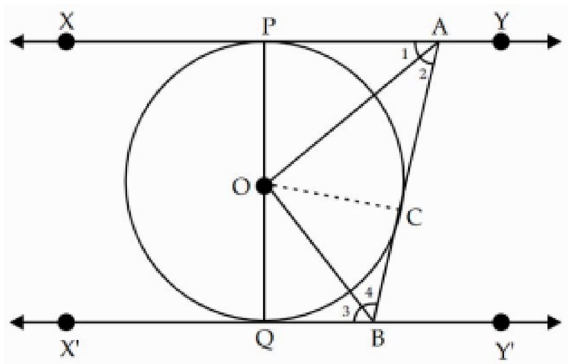
For no solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1} \Rightarrow \frac{3}{2k-1} = \frac{1}{k-1} \Rightarrow 3k - 3 = 2k - 1 \Rightarrow k = 2$$

SECTION C

26	**Ans:** Let $\sqrt{5}$ is a rational number then we have $\sqrt{5}=\frac{p}{q}$, where p and q are co-primes. $\Rightarrow p=\sqrt{5}q$ Squaring both sides, we get $p^2=5q^2$ $\Rightarrow$ p^2 is divisible by 5 $\Rightarrow$ p is also divisible by 5 So, assume p = 5m where m is any integer. Squaring both sides, we get $p^2 = 25m^2$ But $p^2=5q^2$ Therefore, $5q^2 = 25m^2 \Rightarrow q^2 = 5m^2$ $\Rightarrow$ q^2 is divisible by 5 $\Rightarrow$ q is also divisible by 5 From above we conclude that p and q have one common factor i.e. 5 which contradicts that p and q are co-primes. Therefore, our assumption is wrong. Hence, $\sqrt{5}$ is an irrational number.	
27.	Ans: Given, $f(x)=x^2-2x-8$ The zeroes of f(x) are given by, f(x) = 0 $\Rightarrow$ $x^2+2x-4x-8=0 \Rightarrow$ $x(x+2)-4(x+2)=0 \Rightarrow$ $(x+2)(x-4)=0$ $\Rightarrow$ $x=-2$ (or) $x=4$ Hence, the zeros of $f(x)=x^2-2x-8$ are $\alpha=-2$ and $\beta=4$ $\alpha+\beta=-2+4=2=\frac{-b}{a}=2$ $\alpha\beta=-2 \times 4=-8=\frac{c}{a}=-8$	
28.	Ans: Let the tens digits and unit digit of the number be x and y respectively. Then, the number will be 10x + y Number after reversing the digits is 10y + x According to the question, x + y = 9... (i) 9(10x + y) = 2(10y + x) $\Rightarrow 88x-11y=0$ $\Rightarrow 8x+y=0$... (ii) Adding equation (i) and (ii), we get $9x=9 \Rightarrow x=1$ Putting the value in equation (i), we get y = 8 Hence, the number is 18. 0R Ans: Let x be the number of right answers and y be the number of wrong answers. According to the question, 3x – y = 40 ….(i) and, 2x – y = 25 ….(ii) On subtraction, we get: x = 15 putting the value of x in (i), we get 3 (15)-y = 40 y = 5 Number of right answers= 15 answers Number of wrong answers= 5 answers. Total Number of questions = 5 + 15 = 20	

29.	Ans: L.H.S = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$ $=\sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$ $= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2\sin A \times 1/\sin A + 2\cos A \times 1/\cos A$ Since, $(\sin^2 A + \cos^2 A = 1)$ $(\sec^2 A = 1 + \tan^2 A, \operatorname{cosec}^2 A = 1 + \cot^2 A)$ $= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$ $= 7 + \tan^2 A + \cot^2 A$ = RHS	
30	Ans: Given, To Prove, Constructions and Figure – 1½ marks Correct Proof – 1½ marks **OR** In the figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and X'Y' at B, what is the measure of $\angle AOB$.  **Ans:** Join OC. Since, the tangents drawn to a circle from an external point are equal. $\therefore$ AP = AC	



In Δ PAO and Δ AOC, we have:
AO = AO [Common]
OP = OC [Radii of the same circle]
AP = AC
$\Rightarrow \Delta PAO \cong \Delta AOC$ [SSS Congruency]
$\therefore \angle PAO = \angle CAO = \angle 1$
$\angle PAC = 2\angle 1$ ...(1)
Similarly $\angle CBQ = 2\angle 2$ ...(2)
Again, we know that sum of internal angles on the same side of a transversal is 180°.
$\therefore \angle PAC + \angle CBQ = 180°$
$\Rightarrow 2\angle 1 + 2\angle 2 = 180°$ [From (1) and (2)]
$\Rightarrow \angle 1 + \angle 2 = 180°/2 = 90°$ ...(3)
Also $\angle 1 + \angle 2 + \angle AOB = 180°$ [Sum of angles of a triangle]
$\Rightarrow 90° + \angle AOB = 180°$
$\Rightarrow \angle AOB = 180° - 90°$
$\Rightarrow \angle AOB = 90°$.

31. **Ans:** (i) Number of outcomes with sum of the numbers 8 = 5
$\therefore$ Required Probability = 5/36
(ii) Number of outcomes with sum of the numbers 7 = 6
$\therefore$ Required Probability = 6/36 = 1/6
(iii) Number of outcomes with sum of the numbers less than or equal to 12 = 36
$\therefore$ Required Probability = 36/36 = 1

32.	Ans: Let days be the original duration of the tour. Total expenditure on tour ₹ 360 Expenditure per day ₹ 360/x Duration of the extended tour (x + 4) days Expenditure per day according to the new schedule ₹ 360/(x + 4) Given that daily expenses are cut down by ₹ 3 As per the given condition, $\frac{360}{x} - \frac{360}{x+4} = 3$ $\Rightarrow 360\left(\frac{1}{x} - \frac{1}{x+4}\right) = 3$ $\Rightarrow \left(\frac{1}{x} - \frac{1}{x+4}\right) = \frac{3}{360} = \frac{1}{120}$ $\Rightarrow \frac{x+4-x}{x(x+4)} = \frac{1}{120} \Rightarrow \frac{4}{x(x+4)} = \frac{1}{120}$ $\Rightarrow x(x + 4) = 480$ $\Rightarrow x^2 + 4x = 480$ $\Rightarrow x^2 + 4x - 480 = 0$ $\Rightarrow x^2 + 24x - 20x - 480 = 0$ $\Rightarrow x(x + 24) - 20(x + 24) = 0$ $\Rightarrow x - 20 = 0$ or $x + 24 = 0$ $\Rightarrow x = 20$ or $x = -24$ Since the number of days cannot be negative. So, x = 20 Therefore, the original duration of the tour was 20 days **OR** Rs.6500 were divided equally among a certain number of persons. Had there been 15 more persons, each would have got Rs.30 less. Find the original number of persons. Ans: Let the original number of persons be x Total money which was divided = Rs. 6500 Each person share = Rs. 6500/x According to the question, $\frac{6500}{x} - \frac{6500}{x+15} = 30$ $\Rightarrow \frac{6500x + 97500 - 6500x}{x(x+15)} = 30$ $\Rightarrow \frac{97500}{x(x+15)} = 30 \Rightarrow \frac{3250}{x(x+15)} = 1$ $\Rightarrow x^2 + 15x - 3250 = 0$ $\Rightarrow x^2 + 65x - 50x - 3250 = 0$ $\Rightarrow x(x + 65) - 50(x + 65) = 0$ $\Rightarrow (x + 65)(x - 50) = 0$ $\Rightarrow x = -65, 50$ Since the number of persons cannot be negative, hence the original numbers of person is 50	

33. **Ans:** Statement – 1 mark
Given, To prove, Construction and figure of 2 marks
Proof of 2 marks

34. Ans: Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere.
Then, the total surface area = CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi\, r\,(h + r)$

$= 2 \times \frac{22}{7} \times 30\,(145 + 30)\ cm^2$

$= 2 \times \frac{22}{7} \times 30 \times 175\ cm^2$

$= 33000\ cm^2 = 3.3\ m^2$

OR

A tent is in shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1m and 4m respectively and the slant height of the top is 2.8m. Find the area of canvas used for making the tent. Also find the cost of canvas of the tent at the rate of 500 per m^2.

Ans: Radius = 2m, Slant height l= 2.8m , height h= 2.1m
Cost of canvas per m^2= Rs.500
Area of canvas used = CSA of cone + CSA of cylinder
$= \pi rl + 2\pi rh$
$= 22/7 \times 2 \times 2.8 + 2 \times 22/7 \times 2 \times 2.1$
$= 17.6 + 26.4$
$= 44 m^2$
Cost of the canvas of tent =44 x 500
=Rs.22,000

35. Ans: For mean , median , mode
To calculate x_i , cumulative frequency , identifying highest frequency
Formulae for mean , median, mode

Mean = $a + \frac{\sum f_i d_i}{\sum f_i} = 135 + \frac{140}{68}$

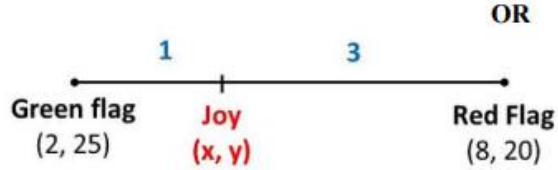
$= 137.05$

Median = $l + \left(\frac{\frac{n}{2} - cf}{f} \times h \right) = 125 + \left(\frac{34-22}{20} \times 20 \right) = 125 + 12 = 137$

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \right) = 125 + \left(\frac{20-13}{40-13-14} \times 20 \right)$

$= 125 + \left(\frac{7}{13} \times 20 \right) = 125 + 10.77 = 135.77$

SECTION E

36.	Ans: (i) Given , a_1 =1000 Common difference, d =100 Total loan= Rs.1,18,000 (ii)$a_{30} = a + 29d$ $= 1000 + 29 \times 100$ $= 3900$ Amount paid in 30[th] installment is Rs.3900 (iii) $S_{30} = \frac{30}{2}$ [2x 1000 +(30 -1) x 100] =15 x 4900 =73,500 Amount paid in 30 installments is Rs.73,500 (OR) The amount he still have to pay after 30 installments=Rs.118000 – Rs. 73,500 =Rs.44,500	
37	Ans: (i) Position of the red flag is $\left(2, \frac{1}{4}\times 100\right) = (2, 25)$ (ii) Distance between the two flags $= \sqrt{(36+25)} = \sqrt{61}\,cm$ (iii) Position of the blue flag $= \left(\frac{2+8}{2}, \frac{25+20}{2}\right)$ $= (5, 22.5)$ **OR**  $x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{1\times 8 + 3 \times 2}{1+3} = \frac{8+6}{4} = \frac{14}{4}$ **= 3.5** $y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1\times 20 + 3 \times 25}{1+3} = \frac{20+75}{4} = \frac{95}{4}$ **= 23.75** Required point is (3.5, 23.75)	

38.	Ans: (i) $\sin 60^0 = \frac{PC}{PA}$ $\Rightarrow \frac{\sqrt{3}}{2} = \frac{18}{PA} \Rightarrow PA = \frac{36}{\sqrt{3}} = 12\sqrt{3}m$ (ii) $\sin 30^0 = \frac{PC}{PB}$ $\Rightarrow \frac{1}{2} = \frac{18}{PB} \Rightarrow PB = 36m$ (iii) $\tan 60^0 = \frac{PC}{AC} \Rightarrow \sqrt{3} = \frac{18}{AC} \Rightarrow AC = 6\sqrt{3}m$ $\tan 30^0 = \frac{PC}{CB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{18}{CB} \Rightarrow CB = 18\sqrt{3}m$ Width AB = AC + CB = $6\sqrt{3}m + 18\sqrt{3}m = 24\sqrt{3}m$ **OR** RB = PC = 18 m & PR = CB = 18√3m $\tan 30^0 = \frac{QR}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{QR}{18\sqrt{3}} \Rightarrow QR = 18m$ QB = QR + RB = 18 + 18 = 36 m. Hence height BQ is 36m.	